2023 Mission Math Utah Spring Competition (6-8)

You will have 40 minutes to complete as much of this test as you can. There are 30 free response questions total, and questions are arranged roughly from easiest to most difficult. Units are not needed. Write answers on the given line below each question. Calculators are not allowed. Do not begin the test until told to do so. Good Luck!

Full Name: _____

Grade: _____

Age: _____

1. Evaluate $7-6+5-4+3-2+1$

2. Joe is 7 less than 3 times Bill's age, and Bill is 3 more than 2 times Steve's age. If Steve is 10, then how old is Joe?

3. Jeffry and Jacob are going trick-or-treating. Every hour, Jeffry can collect 100 pieces of candy and Jacob can collect 75 pieces of candy. In 4 hours, how many more pieces of candy will Jeffry have than Jacob?

4. Julia has sixteen pieces of chocolate. Her older brother has twenty-two more chocolates than her. Julia's younger brother has half as many chocolates as Julia and her older brother together. How many pieces of chocolate does her younger brother have?

5. How many positive integer factors does 24 have?

6. If $(a@b) = 10a + \frac{b}{5}$, what is $((3@2)@5)$?

7. Juan, Jose, and Javier go eat out at a restaurant and decide to split the bill equally. But, Juan and Jose realize that they don't have enough money to split the bill evenly. However, Javier is able to cover for them and he pays \$70, while Juan and Jose pay \$40 and \$25, respectively. When they get home, Juan pays $\$x$ and Jose pays $\$y$ to Javier. What is $x + y$?

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8. Jade is swimming a 200 yard backstroke. She swims her first 50 yards in 27 seconds. For each 50 yards after the first, she is 1 second slower than the previous 50 yards. In seconds, how long does it take Jade to swim her 200 yards backstroke?

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9. What is the sum of all integers between 0 and 100 inclusive that are divisible by both 6 and 8?

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10. What is the 2023^{rd} letter in the string $ABCDE \ldots XYZABC \ldots$, where the string $ABCD \ldots XYZ$ is put front to end an infinite number of times?

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11. There are 10 fruits in a bag: 5 apples, 3 bananas, 2 pears. If two fruits are grabbed from the bag without replacement, find the probability that both fruits are apples.

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12. Evaluate $\frac{7!}{5!} \cdot \frac{4!}{6!} \cdot \frac{5!}{3!} \cdot \frac{2!}{4!}$. Express your answer as a common fraction

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13. James plans to purchase seed to plant pumpkins in a large field. Using a map, he calculates the area to be 180 cm^2. The scale shows 1cm = 20ft. If he plans to cover every 400 ft^2 with one pound of seed, how many pounds of seed will he need to cover the entire field?

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14. At the Adihaya Jayasharmaramankumarguptareddybavarajugopal Elementary School *Avengers: Endgame* musical, adult tickets cost \$10 and tickets for children cost \$5. After the musical, the school raised a total of \$2000 worth of tickets, and a total of 250 people attended. How many of the people who attended were adults?

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15. Jack rolls a standard 6-sided dice and a standard 8-sided dice. What is the probability that the sum of the two numbers he rolls is even?

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16. What is the smallest number greater than 30 that has more factors than 30 does?

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17. If Jasper and Joe weigh a combined 406.6 pounds, Jasper and Jared weigh a combined 394.9 pounds, and Jared and Joe weigh a combined 414.1 pounds, how much do Jasper, Jared, and Joe weigh together? Express your answer as a decimal.

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18. What is the area of a triangle with side lengths 39, 42, and 45?

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19. If Jennifer rolls a standard four-sided dice 5 times what is the probability she gets at least one 3?

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20. What is the smallest possible value of $x + y$ if $xy + x + y = 43$ and x and y are positive integers?

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21. Wilson and Mingchuan are crossing a bridge. They start at the same time, and Mingchuan is running at 3 times Wilson's walking pace. Each time Mingchuan reaches the end of the bridge, he turns around and runs until he meets Wilson, at which he turns around and runs back to the end of the bridge, repeating this process until Wilson crosses the bridge completely. If the length of the bridge is 364.4 smoots, what is the total distance that Mingchuan ran? Give your answer in smoots (a smoot is a unit of measurement).

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22. How many arrangements are there of the word "VESSELS" such that no two S's are consecutive?

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23. In triangle ABC, $\overline{AB} = 7, \overline{BC} = 24$, and $\overline{AC} = 25$. What is the area outside the incircle of ABC but inside $\triangle ABC$? Express your answer in terms of π

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24. Find the last 3 digits of 264^{264}.

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25. The product of 5 consecutive positive integers is equal to 36 times their sum. What is the largest of these integers?

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26. Let A, B, and C be points on a number line with A at 0. If the distance between A and B is 3 and the distance between B and C is 5, find the sum of the absolute values of the possible coordinates of C.

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27. How many ways are there to choose 3 squares in a 4×5 grid such that no two squares are in the same row or column?

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28. Circle ω with radius 5 is centered at the origin. A point (a, b) is chosen on ω such that the slope m between $\left(-\frac{25}{3}, 0\right)$ and (a, b) is maximized. Find the value of $a + b + m$.

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29. In regular hexagon $ABCDEF$ with side length 2, X is the intersection of AC and BE. What is the area of $\triangle FXC$? Express your answer in simplest radical form.

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30. Let $P(x) = x^2 + ax + b$ and $Q(x) = x^3 + bx^2 + ax + 20$. If a and b are non-zero real numbers such that the roots of P are also roots of Q, find the sum of all possible values of $|a| + |b|$ over all possible (a, b).

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