2023 Mission Math Utah Spring Competition (6-8) Theme Round

You will have 20 minutes to complete as much of this test as you can. There are 10 free response questions total, and questions are arranged roughly from easiest to most difficult. Units are not needed. Write answers on the given line below each question. Calculators are not allowed. Do not begin the test until told to do so. Good Luck!

Full Name: _____

Grade: _____

Age: _____

1. Tom caught 1 Pokémon on Monday. Each day after, he caught twice the number of Pokémon as he caught the day before. How many Pokémon did Tom catch in a week?

2. How many ways are there to rearrange the letters in the word "MISTY"?

3. The dimensions of the Pewter City gym are 50 yards by 100 yards. What is the area of the gym in square feet given that the 1 yard is 3 feet?

4. Staryu walks into a restaurant. The restaurant offers 3 different main dishes and 4 different desserts. How many different combinations of exactly 1 dish and any number of desserts (including 0) can Staryu buy?

5. Ash can catch a Pokémon with probability p. When he tries to catch 5 Pokémon, the probability that he catches 2 Pokémon is the same as the probability he catches 3 Pokémon. What is the product of all possible values of p?

6. In the Pokémon world, a year lasts 36 days. What is the minimum number of random people that could be selected to guarantee that at least 11 of them have the same birthday.

7. Through very advanced computer analysis, Ash found out that his Pikachu was at the point (4, 7, 23) in the 3-dimensional battlefield plane. He also discovered that his opponent's Tapu Koko is at the point (1, 19, 19). What is the distance between Pikachu during this moment?

8. When using Thunderbolt, Ash's Pikachu can do 70 damage with 1/4 chance, 80 damage with 1/2 chance, or 90 damage with 1/4 chance. Over the course of 3 moves, what is the probability, expressed as a fraction, that Ash's Pikachu does a total of exactly 220 damage?

9. Brendan sells Treeckos, Torchics, and Mudkips for 300 Pokédollars each at his shop. May visits Brendan's shop with 900 Pokédollars. How many combinations of Pokémon can May buy? Note that May doesn't have to spend all her money, she is allowed to buy multiple of the same Pokémon, and she could buy no Pokémon. The order of the Pokémon she buys does not matter.

10. In their evil schemes, Jessie and James plan to build a base in the shape of a hemisphere. They want to build it with the flat face down in a grassy patch that is in the shape of a rhombus. The rhombus has a side length of 100 feet and 2 acute angles of 30 degrees. If Jessie and James build the largest possible base by volume, the volume of their base can be expressed in the form $a \times b^3 \times \sqrt{c} \times \pi$, where a, b, and c are integers and a is not the multiple of any perfect cube and c is not the multiple of any perfect square. What is a + b + c?