

2023 Mission Math Utah Spring Competition (6-8)

You will have 40 minutes to complete as much of this test as you can. There are 30 free response questions total, and questions are arranged roughly from easiest to most difficult. Units are not needed. Write answers on the given line below each question. Calculators are not allowed. Do not begin the test until told to do so. Good Luck!

Full Name: _____

Grade: _____

Age: _____

1. What is the value of $(1 + 2 + 3) + (2 + 3 + 4) + (3 + 4 + 5)$?

2. Carter randomly picks a number between 1 and 10. The number he picks can be multiplied by 3 to get 24. What is Carter's number?

3. A and B are two digits between 0 and 9. If $A + B + 1 = \underline{1A}$ and $B - A = 7$, what is B? (Here, $\underline{XY} = 10X + Y$.)

4. 15 gallons of water leak out of a full fish tank at the aquarium, and now the tank is $\frac{3}{4}$ full. How much water is left in the tank?

5. Alex flips a weighted coin, where the probability of it landing heads is p . If he flips it 3 times, the probability that he sees exactly 3 heads is $\frac{1}{27}$. What is the value of p ? Express your answer as a common fraction.

6. Mason threw a party in his garage, and the party goers have damaged a light bulb so that it flickers once every 15 seconds. How many times does the light bulb flicker over a 1-hour period?

7. 13% of what number is 2.99?

8. Andy is cleaning out his closet. He donates $\frac{1}{6}$ of his shirts and gives $\frac{3}{8}$ of his shirts to his brother. He now has 11 shirts in his closet. How many shirts did his brother get?

9. Find the value of $(3 \times 4 \times 5 \times 6) - (2 \times 3 \times 4 \times 5)$.

10. If the perimeter of a rectangle is 78 centimeters, and the width of the rectangle is twice the length, what is the area of the rectangle in centimeters squared?

11. At a summer camp, there are 200 people attending. Each student has a favorite sport, and it is either soccer, basketball, or tennis. If there are three times as many people who chose soccer than tennis, and there are ten more people who chose soccer than basketball, how many people chose basketball as their favorite sport?

12. Jade drives down to St. George at 60 miles per hour and drives back at 84 miles per hour. Given that the distance to St. George is 840 miles, what is Jade's average speed for the round trip, in miles per hour?

13. Find the number of 4 digit numbers that satisfy the following properties:

- The second digit is even,
- The first digit is odd,
- The fourth digit is at least twice the second digit.

14. Sophie draws 7 straight lines on a piece of paper. What is the maximum number of intersection points between the lines that she can create?

15. How many integers greater than -5 have the property that their square is greater or equal to their cube?

16. A cherry tree increases its height by 20% at the end of every year. What is the largest possible size (in feet) that a cherry tree could be at the beginning of the year, so that it is no taller than 60 feet by the end of the year?

17. Right triangle ABC with right angle at B is inscribed in a circle of radius 13. If side AB is 10, what is side BC?

18. The area of a square is 2, and the area of a rectangle is 4. What is the smallest possible positive difference of their perimeters? Express your answer in simplest radical form.

19. Two intersecting perpendicular lines AC and BD are drawn with length 6. What is the difference between the maximum and minimum possible areas of quadrilateral ABCD?

20. How many positive integers less than 100 have the property that they are a perfect 4th power, when raised to the 6th power?

21. Joseph wants to draw an equilateral triangle, a square, a regular hexagon, and a regular octagon. What is the minimum number of distinct vertices that Joseph will need to draw?

22. Let x and y be integer solutions to the equation $x + xy + y = 71$. What is the maximum possible value of $x + y$?

23. Iman has 10 books on her bookshelf with a page length of 200, 10 books with a page length of 309, and 10 books with a page length of 437. If she read 1983 pages total by reading entire books from her bookshelf, how many books did she read?

24. For how many values of n between 0 and 100 inclusive does $n + 1$ divide $n^2 + 2n + 3$?

25. Let N be the number of ways to put 1,000 indistinguishable balls into 3 boxes. Find the remainder when N is divided by 1003.

26. The students of West High University participate in a survey about their favorite ice cream flavor. $\frac{3}{5}$ of the responses were chocolate, $\frac{3}{10}$ were vanilla, and $\frac{1}{10}$ of the responses were strawberry. After the survey was over, it was found that the students who enjoyed Neapolitan ice cream were included in the totals for all three flavors. When those results are removed completely from the survey, the portion of chocolate answers rose to $\frac{11}{17}$. What fraction of all the students enjoy Neapolitan ice cream?

27. Let $\zeta(n)$ be the number of positive divisors of n . For how many integers $n \leq 100$ does 5 divide $\zeta(n)$?

28. An interesting property of a regular pentagon is that all its diagonals are equal, and in fact, the ratio of the length of the diagonal to the length of the side of the pentagon is the golden ratio $\frac{1+\sqrt{5}}{2}$. The intersections of the diagonals of regular pentagon $ABCDE$ form a smaller regular pentagon. What is the ratio of the area of this regular pentagon to the area of $ABCDE$? Express your answer as a common fraction in simplest radical form.

29. How many ways can 4 distinct integers be selected from the set of the first 10 integers such that the greatest common factor of all 4 integers is 1?

30. Quadrilateral $ABCD$ is constructed by drawing two non-congruent triangles ACB and ACD such that $AB = AD$ and AC bisects angle BCD . If $AC = 30$ and $BC + BD = 48$, what is the area of $ABCD$?
