Mission Math 6-8 grades

February 17, 2021

MCQ

1. What is the value of 2^3?
   (A) 1 (B) 2 (C) 4 (D) 6 (E) 8

2. What is the value of $\frac{1}{3} + \frac{3}{5}$?
   (A) $\frac{2}{3}$ (B) $\frac{4}{15}$ (C) $\frac{11}{15}$ (D) $\frac{14}{15}$ (E) 1

3. Ben hiked a trail in 15 minutes. Bert hiked the same trail in 20 minutes. If the trail is a half-mile long, how much faster in miles per hour does Ben hike?
   (A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) 1

4. Let $X \,\delta\, Y = X^2 + Y$. What is the value of $(4 \,\delta\, 6) + (2 \,\delta\, 3)$?
   (A) 21 (B) 27 (C) 29 (D) 51 (E) 65

5. Eva spent \$33 on 3 dresses and 2 pairs of socks. Each dress cost \$8.50 more than one pair of socks. What was the cost of each dress?
   (A) \$8.50 (B) \$9 (C) \$10 (D) \$15 (E) \$17

6. Dana drives 21 miles south and x miles east from a shop to her house. If she could just drive straight from the shop to her house, it would be 35 miles. What is x?

   (A) 14 (B) 21 (C) 24 (D) 28 (E) 35

7. What is the last digit of 3^{1000} when expressed in base 10?

   (A) 1 (B) 3 (C) 6 (D) 7 (E) 9

8. A square with side length 1 is inscribed in a circle. What is the area of this circle?

   (A) $\frac{\pi}{2}$ (B) $\frac{2\pi}{3}$ (C) 3 (D) π (E) $\sqrt{2}\pi$

9. Annie has three fair 6-sided dice, where the probability that the sum of the numbers is equal to 8 is $\frac{7}{72}$. What is the probability that the sum of the numbers is equal to 13? Express your answer as a common fraction.

   (A) $\frac{7}{144}$ (B) $\frac{7}{72}$ (C) $\frac{5}{36}$ (D) $\frac{13}{36}$ (E) $\frac{1}{2}$

10. What is the number of non-negative integer solution pairs (A, B), to the following equation:

$$4A + 16B = 256$$

    (A)14 (B) 15 (C) 16 (D) 17 (E) 18

11. It takes Evan a random amount of time between 1 minute and 2 minutes to fold a paper crane. If Evan folds 2 paper cranes, what is the probability that it takes more than 3 minutes?

    (A) $\frac{1}{3}$ (B) $\frac{2}{5}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) 1

12. 3 equilateral triangles with side lengths of 2 are placed side by side to form a trapezoid. What is the ratio of the base of the trapezoid to the height of the trapezoid? Assume that the base is the longer side of the trapezoid.

    (A) $2 : \sqrt{3}$ (B) $4 : \sqrt{3}$ (C) $5 : 2$ (D) $\sqrt{3} : 5$ (E) $4 : 2\sqrt{3}$

13. What is the area of regular octagon with side length of 5 cm?

    (A) $25\sqrt{2}$ (B) $25 + 25\sqrt{2}$ (C) $50 + 25\sqrt{2}$ (D) $25 + 50\sqrt{2}$ (E) $50 + 50\sqrt{2}$

14. If the roots of the cubic $x^3 + 2x^2 - 43x + 40 = 0$ are a, b, and c. Find $a + b + c$.

    (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

15. Define $f(x)$ for a positive integer domain to be $f(x + y) = f(x)f(y)$ where $f(1) = 2$. What is the value of $f(30)$?

    (A) 30 (B) 31 (C) 60 (D) 2^{30} (E) 30^{30}

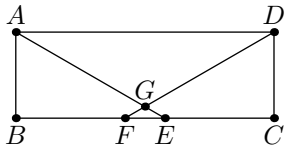
16. Zach picks a random real number x between 0 and 1000. What's the probability that $\lfloor\sqrt{x+\lfloor x\rfloor}\rfloor < \lfloor\sqrt{2x}\rfloor$?

    (A) 0 (B) $\frac{1}{125}$ (C) $\frac{11}{1000}$ (D) $\frac{2}{125}$ (E) 1

17. There are 10 students that Marcus needs to divide into 8 breakout rooms with two having 3 students each and the other two having 2 students each. If all of the breakout rooms of the same size are indistinguishable, how many ways can Marcus make the groups?

    (A) 1575 (B) 3150 (C) 5000 (D) 6300 (E) 12600

18. Rectangle $ABCD$ as shown below has side length $AB = 1$ and $BC = 3$. The point E is drawn on line segment BC such that $\angle BAE = 60^\circ$ and point F is drawn on line segment BC such that $\angle DFC = 60^\circ$. Let G be the intersection of $\overline{AE}$ and $\overline{DF}$. What is the area of triangle EFG?



    (A) $\frac{8\sqrt{3}+3}{24}$ (B) $\frac{1}{16}$ (C) $\frac{7\sqrt{3}-12}{4}$ (D) $\frac{1}{8}$ (E) $\frac{9\sqrt{3}-15}{2}$

19. Let $N = 7^5 \times 11^6 \times 13^7 = 1868313608599296859$. Ruth and Sonia each independently choose a random positive factor of N. (Their factors are not necessarily distinct.) What is the probability that the product of their factors is divisible by N?

    (A) $\frac{1}{210}$ (B) $\frac{1}{8}$ (C) $\frac{3}{16}$ (D) $\frac{1}{5}$ (E) $\frac{1}{2}$

20. There are 3 green balls and 10 orange balls. They are ordered randomly what is the probability that there are not two green balls next to each other?

    (A) $\frac{3}{286}$ (B) $\frac{3}{13}$ (C) $\frac{36}{143}$ (D) $\frac{60}{143}$ (E) $\frac{15}{26}$

FRQ

1. What is the value of $x + y$, if the following two equations are true?

$$4x + 5y = 32$$
$$5x + 4y = 31$$

2. What is the least positive integer n such that $\sqrt{24n + 1}$ is a composite integer?

3. Nina has cartons of 12 eggs and cartons of 7 eggs. If Nina has a sufficient amount of both sizes and can put some number of each in a bowl, What is the least number of eggs that she cannot put in the bowl?

4. Brian's 4 test scores have an average of 72, a mode of 70, and a range of 30. What is Brian's highest test score?

5. How many three-digit numbers are not multiples of 7?

6. Two real numbers, a and b, are chosen randomly and independently in the interval [0,1]. The probability that $a < 1 - b$ AND $a > 2b$ is $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $a + b$.

7. A player chooses one of the numbers 1 through 4. After the choice has been made, two regular four-sided (tetrahedral) dice are rolled, with the sides of the dice numbered 1 through 4. If the number chosen appears on the bottom of exactly one die after it is rolled, then the player wins \$1. If the number chosen appears on the bottom of both of the dice, then the player wins \$2. If the number chosen does not appear on the bottom of either of the dice, the player loses \$1. The expected return is $-\frac{a}{b}$ dollars, where a and b are relatively prime positive integers. Calculate $a+b$.

8. Susan, Anthony, and Charles are trying to guess a number 1 through 5. Susan goes first, Anthony goes second, and Charles goes third. After each number is guessed either they are told that they are right and they win or everyone is told whether the number is larger or smaller than their guess. If all three play with the optimal strategy and the number is 3, the probability that Susan guesses it correctly is $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $10a+b$.

9. Point O has coordinates $(0,0)$ and P has coordinates (0.3,0.4). Points X and Y are on the x- and y-axes, respectively, such that P is on $\overline{XY}$ and $\overline{XY}$ is perpendicular to $\overline{OP}$. Points A and B have the same x-coordinate as X, $\angle AOB$ is right, and B is on ray $\overrightarrow{OP}$. Similarly, C and D have the same y-coordinate as Y, $\angle COD$ is right, and C is on ray $\overrightarrow{OP}$. What is

$$\frac{1}{OA^2}+\frac{1}{OB^2}+\frac{1}{OC^2}+\frac{1}{OD^2}?$$

10. What is the least integer n such that $\lfloor n^2+\frac{250}{n}\rfloor+\lfloor n+\frac{500}{n^2}\rfloor$ is a positive multiple of 25?

1 Answers

1. E
2. D
3. B
4. C
5. C
6. D
7. A
8. A
9. B
10. D
11. C
12. B
13. E
14. A
15. D
16. C
17. D
18. C
19. C
20. E

1. 7
2. 2
3. 65
4. 89
5. 772
6. 7
7. 17

8. 38
9. 4
10. -249