2023 Mission Math Utah Spring Competition (3-5)

You will have 40 minutes to complete as much of this test as you can. There are 30 free response questions total, and questions are arranged roughly from easiest to most difficult. Units are not needed. Write answers on the given line below each question. Calculators are not allowed. Do not begin the test until told to do so. Good Luck!

Full Name: _____

Grade: _____

Age: _____

1. Evaluate $7 - 6 + 5 - 4 + 3 - 2 + 1$

2. Compute $3 \times 5 \times 8$

3. Lavanya made 6 batches of snicker-doodle cookies with 10 cookies per batch for a birthday party, but her hungry friend Agatha ate 2 of the batches. To make up for the eaten cookies, Lavanya made 3 more batches with 10 cookies per batch. How many cookies does Lavanya have now?

4. Find the sum of all perfect squares less than 20.

5. Find the smallest, positive four-digit multiple of 15.

6. Mingchuan and David each have one piece of licorice. Mingchuan eats some of his licorice, and has 3 inches of candy left, while David eats some of his licorice, and has 8 inches of candy left. What is the difference in inches of the amount of licorice the two have left.

7. If $4x + 2 = 4$, what is the value of $4x$ - 6?

_____

8. I have ten pieces of chocolate. Each piece of chocolate is square-shaped with a side length of 5 inches. What is the total area of all the chocolate pieces combined in square inches?

_____

9. What is the measure of an interior angle of a square?

_____

10. The probability that Andy will lose a game is $\frac{1}{9}$. If it is impossible to tie, what is the probability that Andy will win the game? Express your answer as a common fraction.

_____

11. A palindrome is a number that is the same when read forwards and backwards, such as 121. How many integer palindromes are between 10 and 500?

_____

12. Chloe and Marla are playing basketball. Marla scores 6 two pointers and 2 less three pointers than Chloe, who makes 5 two pointers. If their combined point total is 40, how many three point shots did Marla make?

_____

13. What is the units digit of 2023^{3^2}?

14. David and Juntian are playing a card game. Juntian randomly draws a card from a deck of cards numbered 1-10. David tells him that the card's value is even, but Juntian does not know if David is telling the truth. What is the probability that Juntian guesses the parity of the card's value correctly? (Parity is whether the number is even or odd)

15. When rolling 3 fair, standard six-faced dice, what is the probability of rolling the same number on all three dice? Express your answer as a common fraction.

16. Convert $0.\overline{259}$ to a fraction.

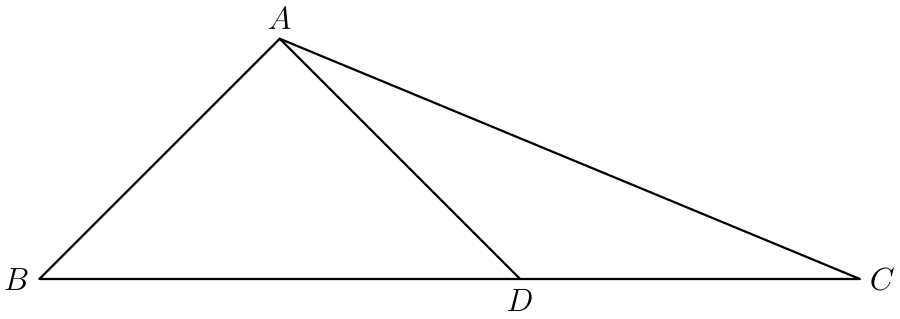
17. John is playing Wheel of Fortune. The wheel has a 1/10 chance of rolling on bankrupt, a 8/10 probability of getting a small amount of money, and a 1/10 change of getting a Jackpot. John has already played 6 times, and his spin landed on cash every time. What is the probability that his next spin gets him a bankrupt or a Jackpot?

18. The remainder when the product $1996 \cdot 2001 \cdot 2014 \cdot 2016$ is divided by 5 is?

19. The sum of two numbers x and y is 798, and the value of the fraction $\frac{x}{y}$ is 0.9. What is the value of $y - x$?

_____

20. In the figure, $BA = AD = DC$ and point D is on segment BC. The measure of angle ACD is 30 degrees. What is the measure of angle ABC?



_____

21. Simplify $\sqrt{32} + \sqrt{18}$

_____

22. What is the smallest integer divisible by 8, 18, and 28?

_____

23. The integer 81 can be written as the sum of smaller perfect squares in a variety of ways. One such way includes five terms: $36 + 16 + 16 + 9 + 4$. If each term has a value between 0 and 81, what is the fewest number of perfect square terms smaller than 81 that can be added together for a sum of 81?

_____

24. Kevin picks a random positive integer x, while Sreeram picks a random positive integer y. Given that $LCM(x, y) = 1012$ and $GCD(x, y) = 2$, what is xy?

_____

25. The numbers 3, 8, x, 13, 20 have the property that the mean is equal to the median. What is the sum of all possible values of x?

_____

26. Olivia is walking from her house to her friend Agatha's house. There is a 8x2 grid between Olivia and Agatha's houses; this means that if Olivia walks 2 blocks north and 8 blocks east from her house, she will arrive at Agatha's house. How many different routes can Olivia take to get to Agatha's house (assume she is only moving north/up or east/right and can not move through blocks)?

_____

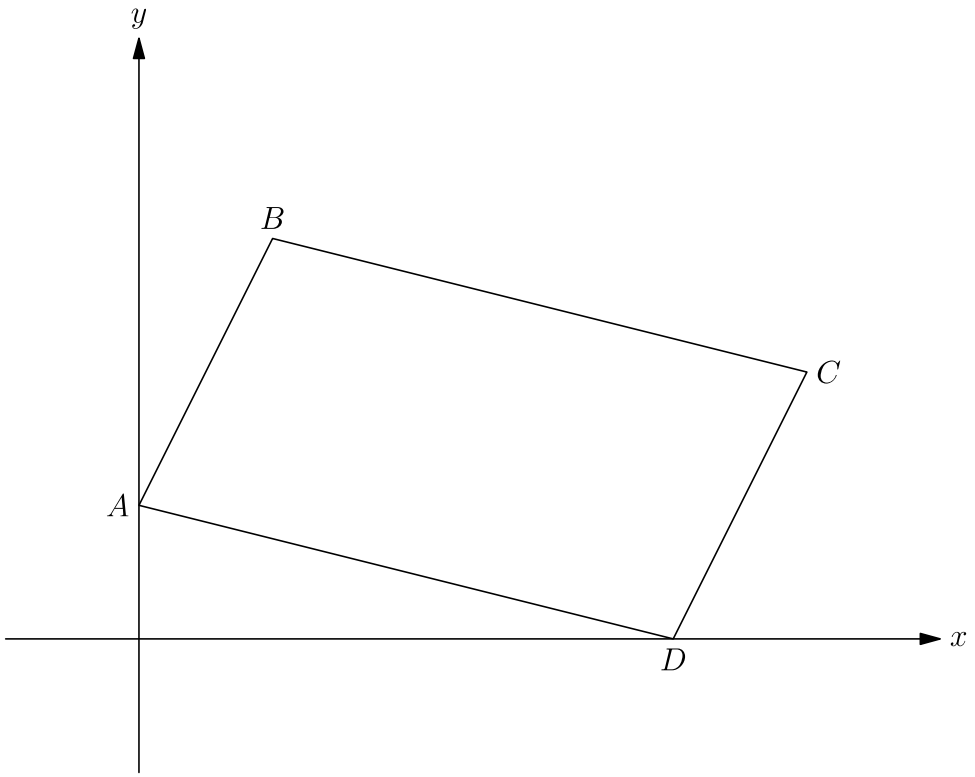
27. There are 300 people in a room and you are interviewing them on what month they were born. What is the maximum number of people you must ask before you come across two different people born in the same month?

_____

28. Rectangle $ABCD$ has side lengths $AB = 21$ and $BC = 28$. There is a point P inside $ABCD$ such that $AP = 10, BP = 17$ and $CP = 25$. Evaluate $BP^2 + DP^2$.

_____

29. In the diagram, the four points have coordinates $A(0,6)$, $B(6,18)$, $C(24,8)$, and $D(15,0)$. What is the area of quadrilateral $ABCD$? (Note that figures are not drawn to scale).



30. Wilson and Mingchuan are crossing the Harvard bridge. They start at the same time, and Mingchuan is running at 3 times Wilson's walking pace. Each time Mingchuan reaches the end of the bridge, he turns around and runs until he meets Wilson, at which he turns around and runs back to the end of the bridge, repeating this process until Wilson crosses the bridge completely. Given that the Harvard bridge is 364.4 smoots, what is the total distance that Mingchuan ran? (Give the answer in smoots as a decimal rounded to the nearest tenth.)