Mission Math Utah Winter Competition (6-8)

You will have 40 minutes to complete as much of this test as you can. There are 25 free response questions total, and questions are arranged roughly from easiest to most difficult. Units are not needed. Write answers on the given line below each question. Calculators are not allowed. Do not begin the test until told to do so. Good Luck!

Full Name: _____

Grade: _____

Age: _____

1. Let ABC be an isosceles triangle with AB=BC. If $\angle ABC = 80°$, what is $\angle BAC$?

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2. Michael swims at a constant rate of 2m/s and Katie swims at a constant rate of 1.5 m/s. In a 1500-meter race, how much time will Michael beat Katie by? Express your answer in seconds.

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3. Define $a \% b$ to be equal to $\frac{a^2-b^2}{a+b}$. Evaluate $(3 \% 4) \% (1 \% 6)$.

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4. Two positive numbers have a product of 48 and a quotient 3. What is their sum?

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5. At the local marketplace, an energy drink costs $1.50. Aidan walks into the store with $20.00, with hopes of buying as many drinks as possible. Disregarding tax, how many drinks can Aidan buy?

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6. Luke and Zach are traveling from a hotel to the airport. Luke decides to wait for the bus at the hotel, while Zach decides to run to the airport. After Zach leaves, Luke waits for 9 minutes before the bus arrives, and he reaches the airport in 4 minutes riding the bus. Two minutes later, Zach arrives at the airport. If the distance from the hotel to the airport is 2 miles, how fast does Zach run (in miles per hour)?

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7. What is the largest 5-digit palindrome divisible by 5? A palindrome is a number that is read the same forwards and backwards, such as 353, 98989, and 84248.

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8. Ronald buys 5 apples and 3 oranges for $23, but he can buy 3 apples and 5 oranges $17. What is the price of 18 apples and 14 oranges?

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9. What is the greatest common factor of $3^5 \cdot 5^3 \cdot 7^6$ and $2^6 \cdot 3^3 \cdot 5^1 \cdot 11^4$?

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10. Todd never responds to emails. On weekdays he gets 20 emails per day on weekdays and 19 emails per day on weekends. How many unread emails will Todd have in 6 weeks?

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11. An airplane can fly from Los Angeles to New York, a 2800 mile journey, in 10 hours with a constant headwind. The return journey with the same tailwind only takes 7 hours. How fast, in miles per hour, can the airplane fly in still air?

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12. In the Martian economy, 7 freckles are worth the same as 3 farkles and 8 farkles are worth the same as 3 furbles. If one US dollar converts to 5 freckles, how many whole $2.00 apples can a martian buy with 4 furbles?

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13. There are 24 permutations of the letters in the word BLUE. For how many of them is the letter E **not** the last letter?

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14. Annabelle draws a circle with radius 12 and center O. Then, she draws equilateral triangle ABO with side length 22. What is the intersection of the area of circle with center O and triangle ABO? Express your answer in terms of pi.

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15. Formula One race cars have tires (tyres) with a diameter of 700 millimeters and their races are 315 kilometers long. If $\pi=3$ in Europe, how many times do the tyres rotate over the course of the race?

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16. A cylinder has diameter of 12 and a height of 16. What is the radius of the smallest sphere that contains this cylinder?

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17. Two fair dice are rolled. What is the probability that their sum is greater than 4? Express your answer as a common fraction.

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18. Player X and player O are playing a game of tic-tac-toe, where player X places 'x' pieces and player O places 'o' pieces. A player wins if they place 3 of their symbol in the same row, column, or diagonal. The current state of the board is shown below. Player X chooses a random location to make their move, and then player O chooses a random location to make their move. What is the probability that player O wins the game? Express your answer as a common fraction. Express your
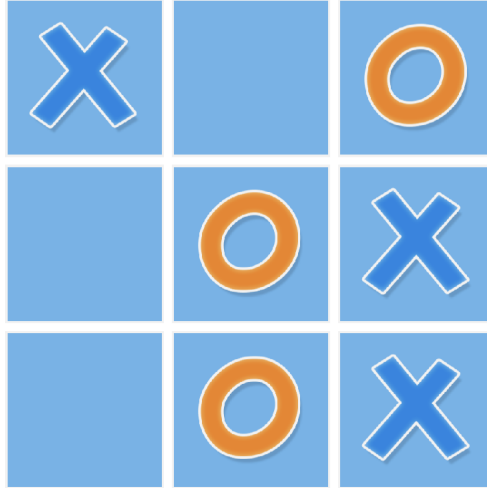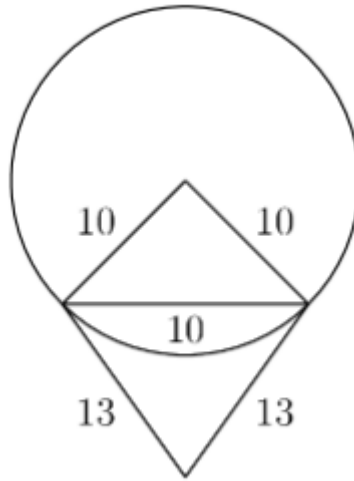

answer as a common fraction.

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19. An unfair coin has a 2/3 chance of landing on heads. If it is flipped 5 times, what is the probability that the number of heads flipped is exactly 3? Express your answer as a common fraction.

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20. The graphs of $|x + y| = 2$ and $|x - y| = 3$ form 4 lines in the coordinate plane. What is the area of the region enclosed by these lines?

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21. Mingchuan and David are playing Clash Royale. Mingchuan has a 50% chance to win each game, and David has a 25% chance to win a game. There is a 25% chance that they tie. If they play each other three times, what is the probability that Mingchuan wins more games than David? Express your answer as a common fraction.

22. Fiona is drawing an ice cream cone on a sheet of paper by putting a circle on top of an upside triangle, as shown by the diagram. Two of the vertices lie on the circle's circumference, forming a chord of length 10. The radius of the circle is 10, and the triangle is isosceles with the sides of length 13. What is the area of the ice cream cone? Your answer will be in the form $a + \frac{b\pi}{c} + d\sqrt{e}$, where a,b,c,d and e are integers, b and c are relatively prime, and e is not divisible by the square of any prime number. Find $a + b + c + d + e$.

23. The integer N can be factored as $178 \cdot 68 \cdot 65 \cdot 12 \cdot 361$. What is the ratio of the sum of the odd factors to the sum of the even factors of N? Express your answer as a common fraction.

24. Let regular hexagon $ABCDEF$ have side length 4, and let M be the midpoint of side BC. The area of triangle FMD can be represented as $a\sqrt{b}$, where b is not divisible by the square of any prime number. Find $a + b$.

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25. The polynomial $f(x) = 18x^3 - 25x^2 - 21x - 2$ has roots r, s, and t. Compute $\frac{1}{r^2} + \frac{1}{s^2} + \frac{1}{t^2}$. Express your answer as a common fraction.

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