

Mission Math Utah Winter Competition (6-8)

You will have 50 minutes to complete as much of this non-calculator test as you can. There are 30 free response questions total, and questions are arranged roughly from easiest to most difficult. Units are not needed. All free response answers are whole number answers unless otherwise stated. Write answers on the given line below each question. Please do not begin the test until told to do so.

Full Name: _____

Grade: _____

Age: _____

1. What is $30 * 40 + 50$?

2. James, John, and Jim play a game. James says any number he wants, then the three take turns saying the number one less than the previous one. The order is James, John, then Jim. For example, James would say “seven” followed by John saying “six” followed by Jim saying “five” and James saying “four”. The person who says “zero” wins. James picks a number from 21 to 23 inclusive. What number must James pick to win?

3. Jerry is skip counting by 4's. He begins at 0, then says 4, then 8, and so on. What is the third number that ends in a 2 that he says?

4. What is the value of x , if $2x + \frac{2}{x} = 4$?

5. January 1, 2022 was a Saturday. What is the next year for which January 1 will be on a Saturday?

6. Amrita has five beads, two are green and three are blue. She wants to put them on a string and make a perfectly circular bracelet with identical ends. How many unique ways can she arrange her beads?

7. Let $A = 2019 + 2021 + 2023 + 2025$, and let $B = 2014 + 2018 + 2022 + 2026 + 2030$. What is the value of $\frac{A}{B}$? Express your answer as a common fraction.

8. Two mangoes and one bunch of grapes cost \$7. Three mangoes and two bunches of grapes cost \$11. How much does a single mango cost?

9. Claire has four Harry Potter lego sets with an average of eighty pieces in each set. If the first set has 81 legos, the second has 64, and the third has 92, how many legos does the fourth set have?

10. Meah bought 2022 pencils from her local manufacturer. She decides to sell them in packs. Meah sells all the packs she can. She puts a prime number of pencils in each pack. What is the largest prime number of pencils per pack she can sell so none are left?

11. In rectangle ABCD, $AB = 12$ and $BC = 5$. Point E is inside of the rectangle such the the area of triangle ABE is 24. Find the area of triangle CDE.

12. What is the largest prime number that is 36 less than a perfect square?

13. Find the units digit of $3^{1986} - 2^{1986}$.

14. What is the maximum finite number of intersection points with a triangle and a square?

15. 2 arithmetic sequences have the same first term. If the positive difference between the 10th terms of each sequence is 24, what is the positive difference between the 25th terms of each sequence?

16. $\triangle ABC$ is a right triangle with $\overline{AB} = \overline{AC} = 6$. Points D and E are on side \overline{BC} such that $\overline{BD} = \overline{DE} = \overline{CE}$ and E is between D and C . What is the length of \overline{AD} ? Express your answer in simplest radical form.
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17. Brendon puts candy into a jar for MMU summer campers to try to guess how many candies are in it. There are 60 summer campers total and 90% of them guess a number between 100 and 200, inclusive. 44 guess a number between 100 and 150, inclusive, and 15 guess a number between 150 and 200, inclusive. How many summer campers guess that there are 150 candies in Brendon's jar?
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18. When Callie dumped the coins out of her piggy bank, she noticed that there were only nickels, dimes and quarters. The number of dimes was one more than twice the number of quarters, and there were half as many quarters as nickels. The coins dumped out of Callie's piggy bank had a total value equal to \$10.00. What was the total value of the nickels and dimes dumped out of Callie's piggy bank? Express your answer as a decimal to the nearest hundredth.
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19. m and n are positive integers such that m divided by 9 has a remainder of 3 and n divided by 6 has no remainder. What is the greatest positive integer that mn is necessarily divisible by?
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20. Pencils come in packages with 6 pencils, 9 pencils, and 20 pencils. What is the largest amount of pencils that cannot be purchased?

21. Let $\triangle ABC$ be a right triangle with $\overline{AB} = 10$, $\overline{AC} = 6$, $\overline{BC} = 8$. The triangle is then folded so that point A coincides with point B . What is the length of the crease fold as an improper fraction?

22. A cube with side length 12 is inscribed in a sphere. The surface area of a sphere is found by $4\pi r^2$. How many square inches greater is the surface area of the sphere than that of the cube? Express in terms of π .

23. Two real numbers x and y are selected on the interval $(0, 3)$. As a fraction, what is the probability that a triangle with side length 1, x , y exists? Express your answer as a common fraction.

24. The line $2y + x = 0$ intersects the parabola $y = x^2 - a$ at two points A and B for some positive real number a . If the length of \overline{AB} is exactly 1, what is the value of a ? Express your answer as a common fraction.

25. The graph of the equations $y = \frac{1}{x}$ and $x = y^2 - 9$ intersect at 3 points (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) . What is the value of $x_1 + x_2 + x_3$?
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26. In quadrilateral $ABCD$, $\angle ABC = \angle CDA = 90^\circ$ and $\angle ACD = \angle BAD = 60^\circ$. If $\overline{AB} = \overline{AD} = 1$, what is the value of $\overline{AC}^2 + \overline{BD}^2$? Express your answer as a common fraction.
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27. Mr. Shah has a coin worth 1 cent, two coins worth 2 cents each, three coins worth 3 cents each, four coins worth 4 cents each, five coins worth 5 cents each, six coins worth 6 cents each, and seven coins worth 7 cents each. He gives coins to each of his sons in a way so that each son has the same amount of money. He gives out all his coins. Using this information, what is the largest number of sons that Mr. Shah could have?
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28. Aaron writes the number $30!$ in base 10. Emma computes the sum of every other digit that Aaron wrote down, starting with the leftmost digit. Given that Emma's sum was 53, what is the sum of all of the digits that Aaron wrote down?
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29. Harold runs at a constant speed of 800 meters per every 5 minutes. Jim runs at a constant speed of 700 meters per 4 minutes. Jim and Harold are both on a 400 meter circular track, in the same spot. They start running in opposite directions without pause. After 5 hours, how many times will Jim and Harold run past each other? Assume the very beginning does not count as one of the passes.

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30. In rectangle $ABCD$, there is a point P inside of $ABCD$ such that $\overline{AP} = 39$, $\overline{BP} = 25$, and $\overline{PC} = 52$. Find \overline{DP} .
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