Spring Competition 2022 6-8 Answers and Solutions

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Answers are on the next page, followed by solutions to each problem, one per page.

ANSWER KEY:

1. 15
2. 187
3. 42
4. 9
5. 792
6. 178
7. 8
8. 26
9. 7
10. 12
11. 10
12. 6
13. 15
14. 2475
15. 15
16. 11
17. 100
18. 41
19. 2
20. 3
21. 60
22. 1
23. 13
24. 31
25. 4
26. 20
27. 13
28. 1680
29. 68
30. 21

Problem 1. Natalya wants to visit the fair with her friends Chloe and Shreya. Each ticket costs 5. How much does it cost to buy tickets for herself and her friends?

In total there are three people (Natalya, Chloe, and Shreya). If each ticket cost 5 dollars and there are 3 people in total, it will cost $5 + 5 + 5 = \boxed{15}$ dollars to pay for the tickets.

Problem 2. What is the value of $89 + 98$?

$$\begin{array}{cccc} & & {\scriptstyle 1} & \\ & & 8 & 9 \\ + & & 9 & 8 \\ \hline & 1 & 8 & 7 \end{array}$$

We have $9 + 8 = 17$, so the 7 goes down and the 1 is carried over above the 8. Then $1 + 8 + 9 = 18$, which we put at the bottom. So the answer is $\boxed{187}$.

Problem 3. I have 3 special 4-sided dice and 5 regular 6-sided dice. How many faces do these eight objects have altogether?

Each 4-sided dice have 4 faces and each 6-sided dice has 6 faces. This means that the total number of faces is $3 \cdot 4 + 5 \cdot 6 = \boxed{42}$.

Problem 4. A rectangle and a square have the same area. The perimeter of the square is 24, and the perimeter of the rectangle is 26. What is the length of the longest side of the rectangle?

The perimeter of a square is the sum of all the four side lengths. All of the side lengths of a square are equal. So the side length of the square is $\frac{24}{4} = 6$. The area of this square is $6^2 = 36$.

The rectangle must also have an area of 36. The area of a rectangle is its base times its height so its base times its height has to be equal to 36. The perimeter of the rectangle is 26 so the base plus the height must equal to $\frac{26}{2} = 13$. Testing numbers for the base and height yields that the base and height are 4 and 9 so the longest side of the rectangle is $\boxed{9}$.

Problem 5. From all whole numbers between 1 and 1000 whose digits add to 8, the smallest and largest numbers are chosen. What is the difference between these two numbers?

To find the largest number with sum of digits 8, we have to understand that a number will be automatically larger than another number if it has more digits than it. The most digits a number between 1 and 1000 can have is 3 digits so our largest number will be a 3 digit number. We also want to make sure that the hundreds digit of our number is the largest possible. We can put 8 in the hundreds place. In order for the number to have sum of digits of 8, we have to put 0s as the other digits so our largest number will be 800.

The smallest number that will have sum of digits of 8 is simply just 8 itself because for all the number smaller than it (1, 2, 3, ..., 7), the sum of digits is less than 8. So the answer is $800 - 8 = \boxed{792}$.

Problem 6. The parking lot outside of the University of Utah Obert C. and Grace A. Tanner humanities center has 10 rows of 20 parking spots each. If 22 Mission Math staff members were the only ones to park in the parking lot, how many empty spots are in the parking lot?

There are a total of $10 \cdot 20 = 200$ empty parking spots in the parking lot. Once 22 Mission Math staff members park their cars, there will be $200 - 22 = \boxed{178}$ empty parking spots left.

Problem 7. Lasya wants to travel to Hawaii for spring break, but she lives 4000 miles away. She flies to Hawaii in a plane moving at 500 miles per hour. How many hours is her flight?

Distance = rate x time. Lasya will travel a total distance of 4000 miles. The rate she is going at is 500 miles/hour. We now have the equation $4000 = 500 \cdot \text{time}$. Dividing both sides by 500, we get time = $\boxed{8}$ hours.

Problem 8. If you add the number of faces, vertices, and edges on a cube, what is the total?

A cube has 6 faces, 8 vertices, and 12 edges so the answer is $6 + 8 + 12 = \boxed{26}$.

Problem 9. How many positive multiples of 3 less than 100 include the digit 3?

There is only one 1-digit multiple of 3 that contains 3, which is 3.

For a two digit number to contain a 3, it must either have a 3 in the first digit or a 3 in the second digit. If 3 is the first digit, the possibilities are $30, 33, 36, 39$. If 3 is the second digit, the possibilities are $33, 63, 93$.

In total, the only numbers that work are $3, 30, 33, 36, 39, 63, 93$ for a total of $\boxed{7}$.

Problem 10. To order ice cream at Harry's Ice Cream shop, you must choose one flavor and one topping option. The options for flavor are Chocolate, Strawberry, and Vanilla, and the options for toppings are chocolate drizzle, gummies, blueberries, or no toppings. How many different icecream orders can be made?

There are 3 options for flavor and 4 options for toppings. So, there are $3 \cdot 4 = \boxed{12}$ ways to make an ice cream.

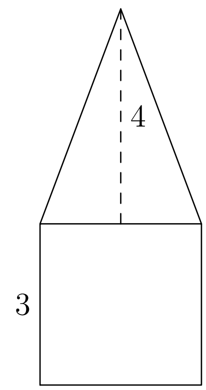
Problem 11. If 5 oranges weigh the same equal 3 apples, and 24 apples weigh the same as 4 watermelons, how many oranges weigh the same as 1 watermelon?

If 24 apples weigh the same as 4 watermelons, then 6 apples weigh the same as one watermelon. Additionally, if 5 oranges weigh the same as 3 apples, then 10 oranges weigh the same as 6 apples. Since 6 apples also weigh the same as a watermelon, the answer is $\boxed{10}$.

Problem 12. Suppose $a\Box b = \frac{8a}{3b}$. What is the value of x if $12\Box(x\Box 1) = 2$?

Let $x\Box 1 = y$. Then $12\Box y = \frac{96}{3y} = 2$, which means $y = 16$. Thus we have $x\Box 1 = \frac{8x}{3} = 16$, which means $x = \boxed{6}$.

Problem 13. An isosceles triangle with a base of 3 and a height of 4 sits directly atop a square, with the base of the triangle coinciding with one side of the square. What is the area of the entire figure?



The square has a side length of 3, so the area of the square is 9. The area of the triangle is $\frac{3 \cdot 4}{2} = 6$. So the area of the entire figure is $9 + 6 = \boxed{15}$.

Problem 14. Boris is buying a bicycle that is originally priced at \$2500, but is on sale for 10% off. Then, a 10% tax applied to the reduced price. How much does Boris pay for the bicycle?

If something is on sale for 10 percent off, it will cost $\frac{9}{10}$ of the original price. So, before tax, the bike costs $2500 \cdot \frac{9}{10} = 2250$ dollars. Applying a 10 percent tax to something multiples its price by $\frac{11}{10}$. So, the bike will cost $2250 \cdot \frac{11}{10} = \boxed{2475}$ dollars after tax.

Problem 15. Jerry can paint a wall in 60 minutes on his own. Ruth can paint that same wall in 20 minutes. Working together, how long would it take for them to paint the wall?

Notice that Ruth paints 3 times as fast as Jerry. So, if Jerry paints an area of A, then Ruth will paint an area of $3A$ after the same amount of time. So, at the end, Ruth will have painted $\frac{3}{4}$ of the wall and Jerry will have painted $\frac{1}{4}$ of the wall. It takes Ruth $\frac{3}{4} \cdot 20 = 15$ minutes to paint $\frac{3}{4}$ of the wall, and Jerry $\frac{1}{4} \cdot 60 = 15$ minutes to paint $\frac{1}{4}$ of the wall. So the answer is $\boxed{15}$.

Problem 16. Alfonso and a group of his friends go out to eat. Each person orders either a salad for \$9.00 or a pasta for \$10.00. They spend a total of \$102.00. How many people (including Alfonso) are in Alfonso's group?

Assume that s people bought salads and p people bought pasta. Then we have $9s + 10p = 102$. Since $10p$ always ends in a 0, $9s$ must end in a 2 so that $9s + 10p$ ends in a 2. This implies that $s = 8$. So, $9 \cdot 8 + 10p = 102$, which means $p = 3$.

So 8 people ordered a salad and 3 people ordered pasta. This means that there are $\boxed{11}$ people in the group.

Problem 17. Mary is taking five tests. On the first four tests, she had an average of 90 points. How many points does Mary need on her fifth test to bring her test average up to 92 points?

Let Mary's last test score be x. Since the average of her first 4 tests was 90, then the sum of her 4 scores on those tests was $90 \cdot 4 = 360$. So the sum of all five of her test scores is $360 + x$.

We require that $\frac{360+x}{5} \geq 92$. This inequality simplifies to $x \geq 100$. So Mary needs a score of $\boxed{100}$ in order to bring her average up to 92.

Problem 18. Mandy's 12 hour digital clock shows $11:20$ right now. In how many minutes will Mandy's clock show the digits $1, 1, 2, 0$ for the first time again, not necessarily in that order?

If there is a time containing 11 as the hour part, then the 2 and the 0 will be in the minutes slots. The only two possible times here are $11:20$ or $11:02$. Both of these are on or before our initial time, so this case doesn't work.

The next best thing is if 12 is the hour part. In that case, we must have 1 and 0 in minute part. The two times satisfying this are $12:01$ and $12:10$. $12:01$ is sooner than $12:10$, so $12:01$ is the first time that we have $1, 1, 2, 0$ in some order after our initial time. $12:01$ is $\boxed{41}$ minutes after $11:20$.

Problem 19. I roll a 6-sided dice 12 times. How many times would I expect to roll a two?

$\frac{1}{6}$ of the time, I roll a 6. So, after 12 rolls, I expect to roll $\frac{12}{6} = \boxed{2}$ 6s.

Problem 20. What is the remainder when the 12 digit number 432143214321 is divided by 6?

If a number is odd, then it must leave a remainder of $1, 3$, or 5 when we divide it by 6. 432143214321 is odd, so we know that the answer must be 1, 3, or 5. If a number is divisible by 3, then it must leave a remainder of 0 or 3 when divided by 6.

Remember that a number is divisible by 3 only if the sum of the digits of that number is divisible by 3. The sum of the digits of 432143214321 is 30, which is divisible by 3. So, 432143214321 is divisible by 3. This means that the answer is 3 or 0.

So, combining this with the condition that the remainder must be 1, 3, or 5, the only possibility for the remainder is $\boxed{3}$.

Problem 21. How many unique ways can I arrange the letters in the word "APPLE"?

Color one of the Ps blue and one of them red. Now, we want to arrange the 5 letters "APPLE." Since all of the letters are different now, the number of ways to arrange them is $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$. However, the Ps shouldn't be considered different.

Notice that the ordering of letters $APPLE$ and $APPLE$ are counted as two different situations in our original count of 120. However, those two situations are the same and should only be counted once. In fact, for any arrangement of the letters, there is another arrangement that we counted in our original total that is identical (switch the colors of the two Ps and you will get it).

So, every arrangement is counted twice in the count of 120. This means that to get everything counted once, we have to divide 120 by 2. So the correct number of arrangements is $\frac{120}{2} = \boxed{60}$.

Problem 22. Gigi is baking a pecan pie for Thanksgiving and needs $\frac{1}{4}$ cups of butter. Each stick of butter has 4 tablespoons. Given that sixteen tablespoons is equal to one cup, how many sticks of butter does Gigi need?

Gigi will need $\frac{1}{4}$ cups $\cdot \frac{16 \text{ tablespoons}}{1 \text{ cup}} = 4$ tablespoons to make her pie. Since each stick of butter has 4 tablespoons, she needs exactly $\boxed{1}$ stick of butter.

Problem 23. If the number T24X8Y is divisible by 4, 5, and 9, what is the largest possible value of $T + X + Y$?

If a number is divisible by 4 and 5, it must be divisible by 20. So, it must end in a 0. This means that $Y = 0$. Additionally, if a number is divisible by 9, then the sum of the digits of that number must be divisible by 9. So, $T + 2 + 4 + X + 8 + 0 = T + X + 14$ must be divisible by 9.

Notice that T and X are both less than 10, so $T + X < 19$. This means $T + X + 14$, a multiple of 9, is less than 33. The only multiples of 9 between 14 and 33 are 18 and 27. These two give $T + X = 4$ or $T + X = 13$. The largest possible value of $T + X$ is 13.

An so, the largest possible value of $T + X + Y$ is $\boxed{13}$.

Problem 24. What is the largest prime number that is a factor of $43^2 - 12^2$?

Recall the identity $a^2 - b^2 = (a+b)(a-b)$. Plugging in 43 and 12 into this, we get $43^2 - 12^2 = (43+12)(43-12) = 55 \cdot 31$. $55 = 5 \cdot 11$, so $43^2 - 12^2 = 5 \cdot 11 \cdot 31$. Clearly the largest prime divisor of this is $\boxed{31}$.

Problem 25. Of all possible rearrangements of the numbers 1, 2, 3, 4, 5, how many of them have the property that the sum of the first three numbers is 9, and the sum of the middle three numbers is 12?

The only way for the sum of the middle 3 numbers to be 12 is if they are $3, 4, 5$. We will do casework based on what the value of the 4th number is.

Case 1: 4th number is 5. Then we must have _345_ or _435_ as our sequence. Since the sum of the first 3 numbers is 9, this means that the first blank has to have a 2. The only number remaining for the last blank is 1, giving us two sequences: 23451 or 24351.

Case 2: 4th number is 4. Then our sequence must be _534_ or _354_. In order for the sum of the first 3 numbers to be 9, the first number must be 1. This means that the last number must be 2, giving us the two sequences 15342 and 13542.

Case 3: 4th number is 3. Then the sequence must be _543_ or _453_. This implies that the first number must be 0, which doesn't work. So there are no sequences in this case.

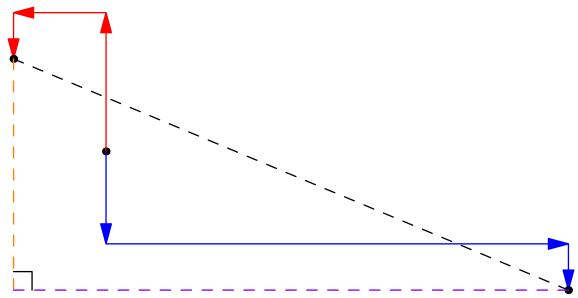
In total, we have $\boxed{4}$ sequences that work.

Problem 26. Gary has a mixture of 25% milk and 75% water, mixed thoroughly. He pours some of it out, and replaces all of that with milk. After doing that, he has exactly 60% water in his mixture. What percent of the total volume did he pour out at the beginning?

When we remove some amount of water, say x percent, then we must also remove $\frac{x}{3}$ percent milk, because the mixture is thoroughly mixed and $\frac{75}{25} = 3$.

We must have poured out exactly 15 percent water in order to reduce it down to 60%. Removing 15 percent water means that we also have to remove $\frac{15}{3} = 5$ percent milk. In total, we have to remove $15 + 5 = \boxed{20}$ percent of the mixture at the start.

Problem 27. Maya and Jagger are at school together. They leave school and head home. Maya travels three miles north, two miles west and then one mile south. Jagger travels two miles south, ten miles east, and then one mile south. How many miles away from each other are they?



Maya's path is shown in red above, and Jagger's path is blue.

The length of the blue segment going East has length 10, as given in the problem. The length of the red segment going West has length 2. So, the total length of the purple segment is $10 + 2 = 12$.

Notice that the total distance that Jagger travels South is $2 + 1 = 3$. The total distance that Maya travels North is $3 - 1 = 2$. This means that the length of the orange segment is $2 + 3 = 5$.

By the Pythagorean theorem, the length of the dotted black segment must be $\sqrt{5^2 + 12^2} = \boxed{13}$.

Problem 28. Jeffery has 9 different colored pens. He wants to give all of his pens to 3 kids, so that each kid gets 3 pens. How many ways are there for him to do this?

Let the pens be colored $A, B, C, D, E, F, G, H, I$, and J. Consider rearranging those pens in a line. Then, the first kid will get the first 3 pens, the second kid will get the middle 3 pens, and the third kid will get the last 3 pens. For example, consider the following rearrangement.

$$\underbrace{\text{B D E}}_{\text{Kid 1}}\underbrace{\text{A G I}}_{\text{Kid 2}}\underbrace{\text{H J F}}_{\text{Kid 3}}$$

There are $9! = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$ ways to line the pens up in a row. However, giving pens B, D, and E to Kid 1 is the same as giving them pens D, B, and E.

There are 6 possible ways to rearrange the 3 pens that a person gets. There are 3 kids, so there each possible way to distribute pens to the kids is counted $6 \cdot 6 \cdot 6 = 216$ times. So we have to divide our initial count of $9! = 362880$ by 216 to get the answer. 362880 divided by 216 is $\boxed{1680}$.

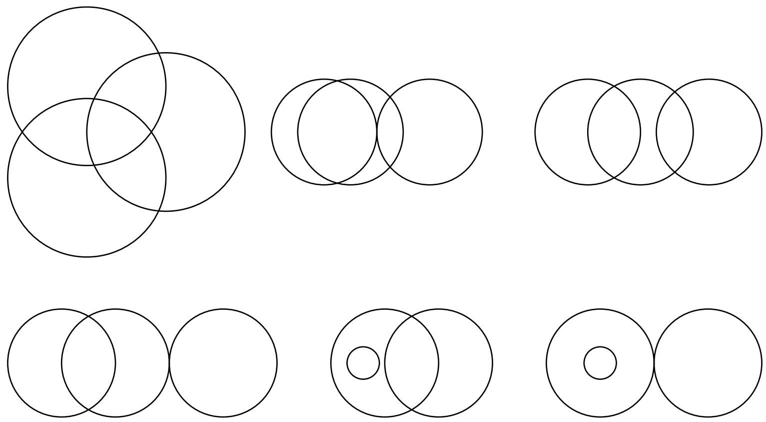
Problem 29. A snowstorm deposited 16.2 inches of snow on the ground. If the snowplows take 42 minutes to clean 10 inches of snow off roads, and the rate of snow plowing remains constant, how long will it take, to the nearest minute, to clean the roads after this storm?

Notice that the snowplow takes 4.2 minutes to clear 1 inch off of the ground. So, it will take $16.2 \cdot 4.2$ minutes to clean of 16.2 inches. $16.2 \cdot 4.2 = 68.04$, which rounded to the nearest integer is $\boxed{68}$.

To compute $16.2 \cdot 4.2$ easily, we can write it as $(16 + 0.2)(4 + 0.2) = 16 \cdot 4 + 0.2 \cdot 16 + 0.2 \cdot 4 + 0.2 \cdot 0.2 = 64 + 0.2 \cdot 20 + 0.04 = 68.04$.

Problem 30. Three different circles are drawn in the plane. There are exactly S points that are on 2 or more of the circles. What is the sum of all possible values of S?

Two circles can intersect at most twice. So 3 circles can intersect at most 6 times. So the only possible values of S are $6, 5, 4, 3, 2$, or 1.



Demonstrations are shown for all 6 possible values above. This means that the answer is $6+5+4+3+2+1 = \boxed{21}$.