Multiple Choice

1. There are 5 girls and 6 boys that are going to be in 3 groups. The groups would be of size 3,4 and 4 with 2 boys in each group. How many distinct ways are there to make the 3 groups?

   (a) 750 (b) 2700 (c) 450 (d) 225 (e) 1350

2. How many ways are there to put 6 indistinguishable oranges into 3 different crates so that each crates has at least one orange in it?

   (a) 13 (b) 11 (c) 12 (d) 10 (e) 19

3. On Pi day, Peter the perfect pie baker baked exactly π pies. Peculiarly, all of his pies are perfect cylinders with π radius and π depth. In terms of π, what volume of pie did Pete

   (a) π^5 (b) $2\pi^3$ (c) 4π (d) π^π (e) π^4

4. What is the slope of the line perpendicular to $y = 2x + 87$?

   (a) $\frac{1}{2}$ (b) -2 (c) $\frac{-1}{2}$ (d) 2 (e) 1

5. In how many ways can a teacher separate his 10 students into a group of 4 and a group of 6 if he has to keep two students, Neal and Peter, in separate groups?

   (a) 50 (b) 110 (c) 24 (d) 112 (e) 25

6. How many diagonals of a regular octagon are not parallel to one of the sides?

   (a) 10 (b) 12 (c) 8 (d) 9 (e) 11

7. If AnaMaria was born on a Sunday, July 25th 2003, on what day of the week will she be 1039 days old?

   (a) Thursday (b) Monday (c) Tuesday (d) Wednesday (e) Friday

8. The ratio of the number of boys in a school play to the total number of students in the play is $\frac{7}{10}$. What is the ratio of the number of girls in the play to the number of boys in the play?

   (a) $\frac{3}{10}$ (b) $\frac{3}{7}$ (c) $\frac{4}{7}$ (d) $\frac{1}{10}$ (e) $\frac{3}{10}$

9. Four boys (Albert, Brian, Caleb, and Diego) weigh themselves on a scale. Albert, Brian, and Caleb weigh 391 pounds when on the scale together. Albert, Brian, and Diego weigh 386 pounds on the scale together. Albert, Caleb, and Diego weigh 360 pounds when on the scale together. Brian, Caleb, and Diego weigh 369 when on the scale together. How much does Diego weigh?

   (a) 111 (b) 110 (c) 121 (d) 109 (e) 119

10. How many perfect squares are there between 6^4 and 4^6, excluding both?

    (a) 28 (b) 26 (c) 25 (d) 27 (e) 29

11. How many terminating 0s does 25! have? (Terminating zeroes are those appear at the end of a number)

(a) 5 (b) 2 (c) 6 (d) 8 (e) 7

12. Find the sum of all solutions to the equation $|x - |3x + 10|| = 4x + 8$

(a) 1 (b) $\frac{2}{7}$ (c) $\frac{-11}{7}$ (d) 2 (e) $\frac{1}{3}$

13. The sum of the digits of a six-digit number is 5. What is the product of these digits?

(a) 5 (b) 0 (c) 6 (d) 720 (e) 5

14. Four fair dice are rolled. What is the probability that the product of all the numbers rolled is even?

(a) $\frac{15}{16}$ (b) $\frac{7}{8}$ (c) $\frac{31}{32}$ (d) $\frac{3}{4}$ (e) $\frac{1}{2}$

15. What is the smallest positive integer that when divided by 14, 15, and 35, leaves a remainder of 2?

(a) 37 (b) 2 (c) 212 (d) 1472 (e) 7352

16. There is a 4x4 square with 16 unit squares. How many rectangles are in the figure?

(a) 100 (b) 256 (c) 64 (d) 80 (e) 512

17. For what percentage of the first 100 numbers is the greatest common divisor of that number and 100 equal to 1?

(a) 30 (b) 50 (c) 60 (d) 40 (e) 70

18.

For pigeons, the trait of having a crest is determined by two markers, one inherited from each

(a) $\frac{1}{4}$ (b) $\frac{15}{16}$ (c) $\frac{1}{16}$ (d) $\frac{1}{32}$ (e) $\frac{1}{2}$

19. Tiana plans to compete in a triathlon. She can average 2 miles per hour in the $\frac{1}{2}$ mile swim and 4 miles per hour in the 6-mile run. Her goal is to finish the triathlon in 4 hours. To accomplish her goal, what must her average speed, in miles per hour, be for the 16 mile bike ride?

(a) 3 (b) $\frac{192}{37}$ (c) $\frac{64}{9}$ (d) $\frac{128}{19}$ (e) 11

20. If a and b are positive and unequal, and $a^b = b^a = c$, what is $log_c(a^{2b}b^a)$?

(a) 4 (b) 2 (c) 3 (d) 1 (e) 0

Free Response

1. What is the smallest positive integer greater than 1 that is a perfect square and a perfect cube?

2. Gregory is in a lab and wants to make an acid solution. He has a large amount of a stock of 50% solution and he wants to make as much 30% solution as he can with 200 mL of a 20% solution. What volume of 30% solution can he make?

3. What is $\frac{20!}{17!}$?

4. Find constants A and B such that $\frac{2x}{x^2-5x+6} = \frac{A}{(x-3)} + \frac{B}{(x-2)}$, for x not equal to 2 or 3. What is the sum of A + B?

5. In a group of pandas, the two lightest pandas weigh 21% of the total weight of the group and the three heaviest weigh 63% of the total weight. How many pandas are in the group?

6. On June 28th (2π day), Peter the perfect baker baked P pies. Each pie has I nuts in it. If Peter bakes one more pie with I nuts, then he will have $356-4$P nuts in total. how many different number of pies could Peter have baked, if P and I are both positive integers?

7. There are 10 seats in a Row F at the movie theater. Alice, Bruce, and Candice are sitting in Row F, but each have at least 2 seats between them to avoid getting sick with a virus. How many different ways can they be seated?

8. If $\frac{2020}{n}$ is an integer, what is the minimum absolute value of $2020n+1$?

9. Square ABCD has a point P placed randomly inside its interior. What is the probability that $AP^2+PC^2 \geq BP^2+PD^2$?

10. A circle is drawn inside of a square with side length 2, so that the circle touches the square on all 4 sides. Find the area of the region inside the square, but outside the circle. Please format your answer as $a-b\pi$