
MATHCOUNTS®

2006

■ School Competition ■
Sprint Round
Problems 1–30

Name _____

**DO NOT BEGIN UNTIL YOU ARE
INSTRUCTED TO DO SO.**

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the right-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

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1. What is the value of $5 \times 5 \times 0 + 5 + 5 + 5$?

1. _____

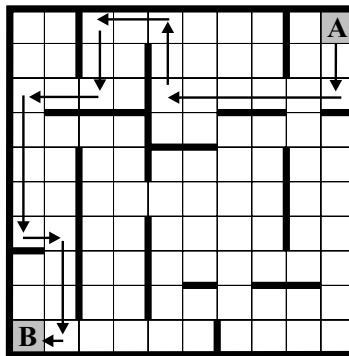
2. Auston is 60 inches tall. Using the conversion 1 inch = 2.54 cm, how tall is Auston in centimeters? Express your answer as a decimal to the nearest tenth.

2. _____ centimeters

3. Julio had four markers and three pencils. He bought the markers for an average of \$0.75 each and the pencils for an average of \$0.50 each. How much did he spend to purchase all seven of these items?

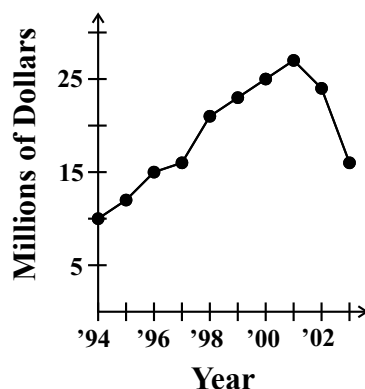
3. \$ _____

4. In the maze to the right, a move is either horizontal or vertical in a single direction. Each move ends when you reach a barrier or the edge of the grid, where you must turn 90 degrees in either direction and then start your next move. You must always remain within the grid. The thicker lines indicate barriers or edges. The 10 moves of one possible route from square A to square B are shown with arrows. What is the least number of moves needed for a route from square A to square B?



4. _____ moves

5. The points on this graph show the end-of-year sales amounts for each year. During what year after 1994 did sales increase the most number of dollars?



5. _____

6. Terry needs at least four square feet of floor space per bird in his chicken coop. Terry plans to build a 30-foot by 20-foot chicken coop. What is the maximum number of chickens he can house?

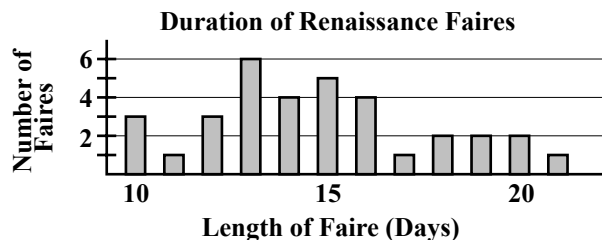
6. _____ chickens

7. Marian wishes to buy a new computer that will cost her \$300. She receives \$5 per hour for watching her younger brother and \$4 per hour for helping her mother with chores. Each week she watches her brother for four hours and helps her mother with chores for 10 hours. How many full weeks must she work to earn enough money to buy the computer?

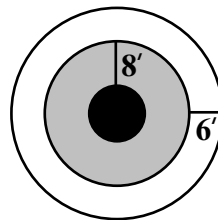
7. _____ full weeks

8. Renaissance faires are festivals where participants dress, talk and act as if they are living in the 1500s. The graph shows the number of days various faires run during their season. For example, three faires run for exactly 10 days. What is the total number of faires that run more than 14 days?

8. _____ faires



9. As shown in the figure to the right, a circular park consists of an outer-ring path for walkers (white) and a ring-shaped flower garden (gray) surrounding a central circular fountain (black). The walking path is six feet wide in all places, the garden ring is eight feet wide in all places, and the fountain has a diameter of 10 feet. What is the diameter of the circle that is the outer boundary of the walking path?



9. _____ feet

10. What is the probability that a positive integer less than or equal to 24 is a factor of 24? Express your answer as a common fraction.

10. _____

11. The measures of the three interior angles of a triangle are 50° , 55° and x° . What is the degree measure of the largest interior angle of this triangle?

11. _____ degrees

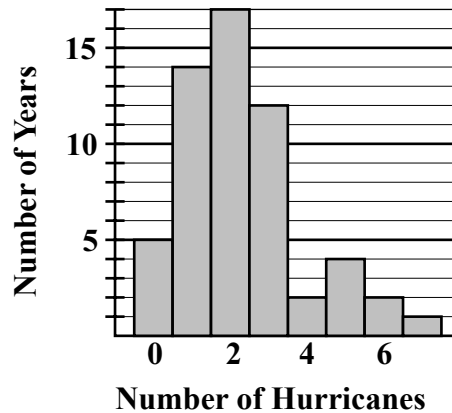
12. The U.S. produces about 5.5 million tons of apples each year. Of the total, 20% is mixed with other products, and the remainder is split equally between apple juice and fresh apple sales. How many million tons of apples are used for apple juice? Express your answer as a decimal to the nearest tenth.

12. _____ million tons

13. A certain organism begins as two cells. Each cell splits and becomes two cells at the end of three days. At the end of another three days, every cell of the organism splits and becomes two cells. This process lasts for a total of 15 days, and no cells die during this time. How many cells are there at the end of the 15th day?

13. _____ cells

14. Using data from 1944 through 2000, the histogram shows the number of years that had a particular number of hurricanes reaching the east coast of the U.S. For example, in 14 of those years there was exactly one hurricane each year that reached the east coast of the U.S. What is the median number of hurricanes per year reaching the east coast from 1944 through 2000?



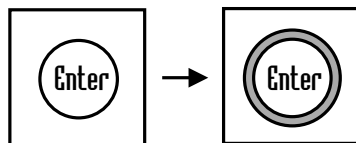
14. _____ hurricanes

15. What is the sum of all positive integers less than 100 that are squares of perfect squares?

15. _____

16. The circular region of the sign (below, left) has an area of 154 square inches. Vanessa would like to place a tiny ribbon (shaded) around the circle's edge. To be sure she has enough ribbon, she decides to buy 2 inches more of the ribbon than the original circle's circumference. How much ribbon will Vanessa need to buy if she estimates $\pi = \frac{22}{7}$?

16. _____ inches



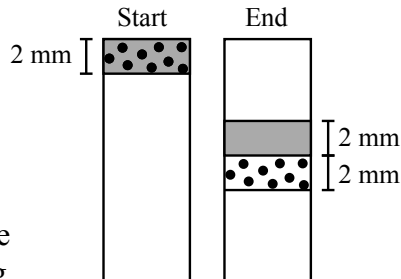
17. When N is divided by 8 the result is greater than 5. What is the least possible integer value of N ?

17. _____

18. Two boys (Jake and Miles) and two girls (Betty and Abby) are elected to the four student council offices (president, vice-president, treasurer and secretary). If a girl is elected president and Jake is elected vice-president, in how many ways can the four students fill the four offices?

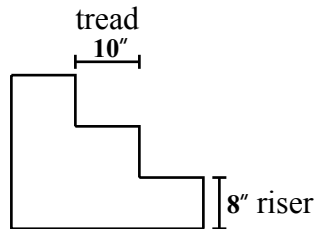
18. _____ ways

19. On a gel, bands are 2 mm wide. Red dye (shaded in picture) travels at 1 mm/minute and yellow dye (dotted in picture) travels at 1.5 mm/minute. At the start, the red and yellow bands are in exactly the same place, making orange (shaded and dotted, left figure). How many minutes will it be until the red and yellow bands no longer overlap, as seen in the right figure?



19. _____ minutes

20. A stairway with three steps has three risers that are each 8 inches high and three treads that are each 10 inches deep. What is the area of this figure that is the side panel of the stairway?



20. _____ sq inches

21. What is the least perfect square which is the sum of three different non-zero perfect squares?
22. One day, 45 frogs were captured from a pond, marked and then returned to the pond. The next day, 40 frogs were observed in the pond, of which 10 had been marked the previous day. Assuming the marked frogs are equally distributed among all the frogs in the pond, what is the best estimate of how many frogs live in the pond?
23. Connie went shopping and spent one-half of her money at Allen's Grocery Store. She then spent one-third of what she had left at Bryan's Bar-B-Q Cafe. After that, she then had \$6 left. How much money did she spend at Allen's Grocery Store?

21. _____

22. _____ frogs

23. \$ _____

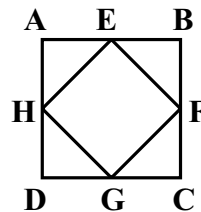
24. Exits on a highway are numbered consecutively from 1 to 50. The distance from exit 41 to exit 50 is 100 km. If each exit is at least 6 km from the next exit, what is the longest possible distance, in kilometers, between any two consecutive exits from exit 41 to exit 50?

24. _____ kilometers

25. What is the coordinate of the point located one-half of the way from $-\frac{1}{2}$ to $1\frac{1}{4}$ on a number line? Express your answer as a common fraction.

25. _____

26. The midpoints of square ABCD are connected to form square EFGH. If $EF = 10$ units, what is the area of square ABCD?



26. _____ sq units

27. A 420-page book contains an average of 600 words per page, and Roslyn read the book at the rate of 360 words per minute. How many hours did it take her to read the book? Express your answer as a mixed number.

27. _____ hours

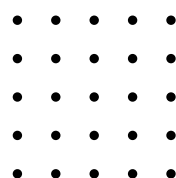
28. An airplane's departure is delayed half an hour, but the flight time is reduced by 10% due to favorable winds. If the flight ends up landing at its destination at the same time it would have landed if there had been no winds and no departure delay, how many hours was the flight originally scheduled to take?

28. _____ hours

29. For any number x , we are told that $x\& = 7 - x$ and $\&x = x - 7$. What is the value of $\&(12\&)$?

29. _____

30. Set R is the set of all possible rectangles such that (1) only the grid points shown here are used as vertices, (2) all sides are vertical or horizontal and (3) no two rectangles in the set are congruent. What fraction of the rectangles in set R are squares? Express your answer as a common fraction.



30. _____