



Mission Math Utah

# Winter Competition

## Division Four: Grades 6-8

December 27, 2019

Name: \_\_\_\_\_

School: \_\_\_\_\_

Grade: \_\_\_\_\_

### General Information

- Do not open the test until you are instructed to do so by the proctor.
- This test contains **35 free response problems**. You will have exactly **60 minutes** to answer them.
- Electronic devices, including calculators, must be turned off and **can not** be used during the test.

### Grading

- Each correct answer is worth 1 point.
- Incorrect or blank answers are worth 0 points. Partial credit will not be given.

### Answers

- Answers must be written clearly in the allotted space. Please write legibly.
- All answers must be expressed in the appropriate form as specified in the question.
- Your answer will be marked incorrect if the graders are unable to read it.

1. Milo is driving his car at 12 miles per hour. If he drives for 6 hours, how many miles does he travel? 1. \_\_\_\_\_ miles
2. A right triangle has a hypotenuse of length 5 inches and a leg of length 3 inches. What is the perimeter of the triangle? 2. \_\_\_\_\_ inches
3. What is the sum of the positive factors of 2019? 3. \_\_\_\_\_
4. Simon's recipe feeds 6 people and requires 2 sticks of butter and 4 cups of flour. He wants to use his recipe to feed 15 people. He needs  $x$  sticks of butter and  $y$  cups of flour. What is the value of  $x + y$ ? 4. \_\_\_\_\_
5. What is the greatest integer  $n$  such that  $n^2 + 10n + 5$  is less than 215? 5. \_\_\_\_\_
6. A set of  $c$  real numbers has an average of 10. If the number 16 is added to the set, the new average of the set would be 11. What is the value of  $c$ ? 6. \_\_\_\_\_

7. Billy has five different-colored pens. If each slot holds one pen, how many ways are there to arrange them in the five slots of his pencil case?

7. \_\_\_\_\_ ways

8. A circle has a circumference of  $32\pi$  inches. What is the area of this circle? Express your answer in terms of  $\pi$ .

8. \_\_\_\_\_ in<sup>2</sup>

9. Sara has 25 cards that are numbered with integers from 1 to 25. She picks one of the cards at random. What is the probability that the number on her card will be a multiple of 2 or 5? Express your answer as a common fraction.

9. \_\_\_\_\_

10.  $a$  and  $b$  are real numbers that satisfy  $4^{4^2+2a} = 2^{2^4+4b}$ . What is the value of  $a - b$ ?

10. \_\_\_\_\_

11. The function  $f(x)$  is defined as  $f(x) = 2x + 3$ , and the function  $g(x)$  is defined as  $g(x) = x^2$ . What is  $f(g(3))$ ?

11. \_\_\_\_\_

12. Mila and Troy are at the starting line of a 400-meter track. They begin running at the same time and in the same direction. Mila runs at a rate of 300 meters per minute, while Troy runs at a rate of 200 meters per minute. After how many minutes will they both be at the starting line again?

12. \_\_\_\_\_ minutes

13. Richard bikes from home to school at an average speed of  $x$  miles per hour. On his way back from school, he walks back on the same route at an average speed of 3 miles per hour. If his average speed for the whole trip was 5 miles per hour, find the value of  $x$ .

13. \_\_\_\_\_

14. The points  $(2,-3)$  and  $(1, 2)$  are vertices of a square. What is the difference between the greatest possible area of this square and the least possible area of this square?

14. \_\_\_\_\_

15. When I arrange  $n$  dots into rows of 5, I have a row of 4 left over. When I arrange the dots into rows of 19, I have 18 left over. If  $n$  is a positive integer less than 150, what is the value of  $n$ ?

15. \_\_\_\_\_

16. How many positive integers  $q$  satisfy the following inequality?

16. \_\_\_\_\_ integers

$$\frac{2019!}{((q!)!)!} > 1$$

17. The product of three distinct positive integers is equal to  $7^3$ . What is the sum of the three integers?

17. \_\_\_\_\_

18. How many people must be in a room to guarantee that at least two of them have the same birthday or at least one of them was born in February?

18. \_\_\_\_\_ people

19. A sphere is placed in a cube such that the cube is tangent to the sphere on all six faces. If the volume of the sphere is  $36\pi \text{ cm}^3$ , what is the surface area of the cube?

19. \_\_\_\_\_  $\text{cm}^2$

20. Two points are chosen at random on a circle, and the line segment connecting them is drawn. What is the probability that the length of this segment is less than or equal to the length of the radius of the circle? Express your answer as a common fraction.

20. \_\_\_\_\_

21. How many ways are there to pick a president, vice president, and three officers from a group of 6 people if each person can hold at most one position?

21. \_\_\_\_\_ ways

22. Two distinct natural numbers are selected from the set  $\{1, 2, 3, 4, 5, 6\}$ . What is the probability that these numbers are coprime? Express your answer as a common fraction.

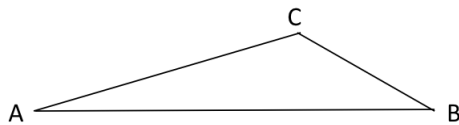
22. \_\_\_\_\_

23. If  $\frac{1}{x} + \frac{1}{y} = 5$  and  $x + y = 15$ , what is the value of  $x^2 + y^2$ ? 23. \_\_\_\_\_

24. For how many integers  $n$  between 1 and 50, inclusive, is it possible to buy  $n$  donuts from a shop which sells only boxes of 6 donuts and 10 donuts? 24. \_\_\_\_\_ integers

25. The parabola defined by the function  $f(x) = x^2 + ax + b$  has its vertex at the point  $(2, 7)$ . What is the value of  $\frac{f(f(f(0)))}{f(0)}$ ? 25. \_\_\_\_\_

26. In  $\triangle ABC$ ,  $\overline{AB}$  has a length of 5,  $\overline{BC}$  has a length of  $\sqrt{5}$ , and  $\overline{CA}$  has a length of  $\sqrt{10}$ . What is the measure of  $\angle C$ ? 26. \_\_\_\_\_ degrees



27. Brian's four test scores have an average of 72, a mode of 70, and a range of 30. What is Brian's highest test score? 27. \_\_\_\_\_

28. Albert and Bethany have the exact same birthday. In the year 2019, Albert is 49 years old and Bethany is 1 year old. How many years will there be, including 2019, such that Bethany's age is a factor of Albert's age? 28. \_\_\_\_\_ years

29. What is the units digit of  $3^1 + 3^3 + 3^5 + 3^7 + \dots + 3^{2019}$ ? 29. \_\_\_\_\_
30. How many factors does the integer  $2019^2 - 1$  have? 30. \_\_\_\_\_ factors
31. The function  $f(x)$  has the property that, for any real numbers  $a$  and  $b$ ,  $f(a + b) = f(a) + f(b)$ . If  $f(1) = -3$ , what is the value of  $f(f(-2))$ ? 31. \_\_\_\_\_
32. A sphere is inscribed in a cube that has a surface area of 24 square meters. A second cube is then inscribed within the sphere. What is the surface area of the inner cube? 32. \_\_\_\_\_  $\text{m}^2$
33. Two real numbers,  $a$  and  $b$ , are chosen randomly and independently between 0 and 1. What is the probability that  $a < 1 - b$  and  $a > 2b$ ? Express your answer as a common fraction. 33. \_\_\_\_\_
34. What is the value of  $\sqrt{\sqrt{7 + 4\sqrt{3}} + \sqrt{7 - 4\sqrt{3}}}$ ? 34. \_\_\_\_\_
35. Let  $a$  and  $b$  be the zeroes of the parabola  $x^2 + 4x + 2$ . What is the value of  $\sqrt{4a + 14} + \sqrt{4b + 14}$ ? 35. \_\_\_\_\_