MATHCOUNTS®

2015
■ Chapter Competition ■
Sprint Round
Problems 1−30

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HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature _____ Date _____

Printed Name _____

School _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

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1. _____ If $\dfrac{a}{b} = \dfrac{3}{5}$ and $b = 10$, what is the value of a?

2. _____ triangles Square ABCD, shown here, has diagonals AC and BD that intersect at E. How many triangles of any size are in the figure?
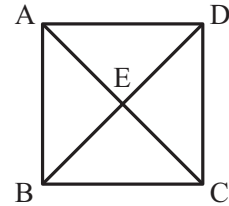
3. _____ When the integers 1 to 100 inclusive are written, what digit is written the fewest number of times?

4. _____ Danica wrote the digits from 1 to 8 across a sheet of paper, as shown, and then circled one digit. If the digits to the left of the circled digit had the same sum as those to the right of the circled digit, which digit did Danica circle?

1   2   3   4   5   6   7   8

5. $ _____ Sally received a sum of money for her birthday. She spent one-third of that money on bus fare to travel downtown. She spent half of the money that remained to treat her best friend to a movie, after which Sally had $12.00 remaining. How much money did Sally receive for her birthday?

6. _____ When six is added to a number, the result is three times the original number. What is the original number?

7. \$ _____ Each time one of Yuan's paintings sells, the gallery sends Yuan a payment equal to 60% of the purchase amount, and the gallery keeps the remainder. If three of Yuan's paintings sold recently for $200, $300 and $500, he should expect to receive three payments totaling how many dollars from his gallery?

8. _____ If x is 15% of 500 and y is 200% of x, what is the value of the sum $x + y$?

9. _____ The square in Figure 1 is cut along its diagonals creating four congruent triangles that then are arranged to create Figure 2. What is the probability that a randomly chosen point within the boundary of Figure 2 is in any of the shaded triangles? Express your answer as a common fraction.
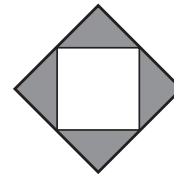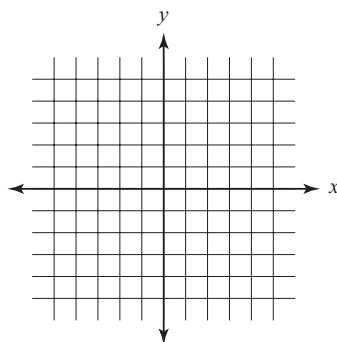
Figure 1   Figure 2

10. _____ If $\sqrt[3]{b} = 5$, what is the value of $2b$?

11. \$ _____ This table shows every category of Martin's monthly expenses. Once completed, it will depict how 100% of Martin's monthly income is budgeted. Based on the information provided, what is Martin's monthly income?

	Rent	Car Loan Payment	Utilities	Food	Gasoline	Savings
Percent of Income	35%	15%	20%	15%		10%
Dollars of Income					$80	

12. _____ units² The points A(−1, 2) and B(3, 2) are graphed on a coordinate plane. Point C is the reflection of point A over the x-axis. What is the area of triangle ABC?

13. _____ What is the greatest prime factor of the product 6 × 14 × 22?

14. _____ If ▲ + ■ = 5 and ▲ − ■ = 3, what is the value of ▲ + ▲?

15. _____ Ms. Spice tells her class that she is thinking of a positive common fraction. The product of the numerator and denominator is 60. When she adds 1 to the numerator and divides the denominator by 2, the resulting fraction is equal to 1. What common fraction is Ms. Spice thinking of?

16. _____ papers Working alone, a professor grades a paper every 10 minutes. The professor spends 20 minutes training an assistant. Then, working together, they grade 2 papers every 15 minutes. For how many graded papers is the amount of time it would take the professor working alone the same as the amount of time it would take the professor and her assistant working together, including the time required for training?

17. _____ If $b = a^2$ and $c = 3b - 2$, what is the product of all values of a for which $b = c$?

18. _____ A set of six distinct positive integers has a mean of 8, a median of 8 and no term greater than 13. What is the least possible value of any term in the set?

19. _____ If $x \uparrow y = (x + y)^2$ for positive integers x and y, what is the value of $(1 \uparrow 2) \uparrow 3$?

20. _____ Segment XY is drawn parallel to the base of triangle ABC. If the area of trapezoid BCYX is 10 units² and the area of triangle AXY is 8 units², what is the ratio of XY to BC? Express your answer as a common fraction.
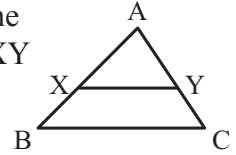
21. _____ What is the greatest possible absolute difference between the mean and the median of five single-digit positive integers? Express your answer as a common fraction.

22. _____ If f is a function such that $f(f(x)) = x^2 - 1$, what is $f(f(f(f(3)))))$?

23. _____ If the sum of an arithmetic progression of six positive integer terms is 78, what is the greatest possible difference between consecutive terms?

24. _____ units² Points A, B and C have coordinates $(-4, 2)$, $(1, 2)$ and $(-1, 5)$, respectively. If triangle ABC is reflected across the y-axis, what is the area of the region that is the intersection of triangle ABC and its reflection? Express your answer as a decimal to the nearest tenth.
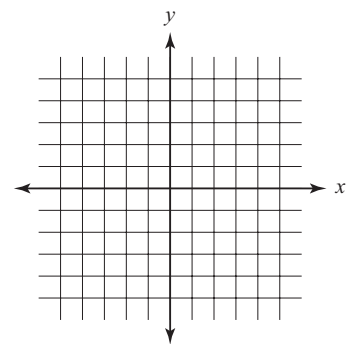
25. _____ On a standard die with six faces, each face contains a different number from 1 through 6. Jake has a non-standard die with six faces, and each face on Jake's die contains an expression with a different value from 1 through 6. In no particular order, the six expressions are $a + 1$, $2a - 5$, $3a - 10$, $b + 8$, $2b + 5$ and $3b + 10$. If a and b are integers, what is the value of the product $a \times b$?

26. _____ For a particular sequence $a_1 = 3$, $a_2 = 5$ and $a_n = a_{n-1} - a_{n-2}$, for $n \geq 3$. What is the 2015th term in this sequence?

27. _____ If Desi flips a fair coin eight times, what is the probability that he will get the same number of heads and tails? Express your answer as a common fraction.

28. _____ ordered pairs How many ordered pairs of integers (x, y) satisfy the equation $x + |y| = y + |x|$ if $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$?

29. _____ When $\dfrac{1}{98}$ is expressed as a decimal, what is the 10th digit to the right of the decimal point?

30. _____ For positive integers n and m, each exterior angle of a regular n-sided polygon is 45 degrees larger than each exterior angle of a regular m-sided polygon. One example is $n = 4$ and $m = 8$ because the measures of each exterior angle of a square and a regular octagon are 90 degrees and 45 degrees, respectively. What is the greatest of all possible values of m?

Forms of Answers

The following list explains acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Judges will score competition answers in compliance with these rules for forms of answers.

All answers must be expressed in simplest form. A "common fraction" is to be considered a fraction in the form $\pm\frac{a}{b}$, where a and b are natural numbers and GCF(a, b) = 1. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not share a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N, a and b are natural numbers, $a < b$ and GCF(a, b) = 1. Examples:

Problem: Express 8 divided by 12 as a common fraction.   *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$

Problem: Express 12 divided by 8 as a common fraction.   *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$

Problem: Express the sum of the lengths of the radius and the circumference of a circle with a diameter

      of $\frac{1}{4}$ as a common fraction in terms of π.   *Answer:* $\frac{1+2\pi}{8}$

Problem: Express 20 divided by 12 as a mixed number.   *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:
Simplified, Acceptable Forms: $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}$, $\frac{\frac{1}{4}}{3}$, 3.5, 2:1

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:
Problem: Evaluate $\sqrt{15}\times\sqrt{5}$.   *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form (\$) *a.bc*, where *a* is an integer and *b* and *c* are digits. The *only* exceptions to this rule are when *a* is zero, in which case it may be omitted, or when *b* and *c* are both zero, in which case they may both be omitted. Answers in the form (\$)*a.bc* should be rounded to the nearest cent unless otherwise specified. Examples:
Acceptable: 2.35, 0.38, .38, 5.00, 5   *Unacceptable:* 4.9, 8.0

Do not make approximations for numbers (e.g., π, $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not perform any intermediate rounding (other than the "rounding" a calculator does) when calculating solutions. All rounding should be done at the end of the computation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \le |a| < 10$, and n is an integer. Examples:
Problem: Write 6895 in scientific notation.   *Answer:* 6.895×10^3
Problem: Write 40,000 in scientific notation.   *Answer:* 4×10^4 *or* 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole number answers should be expressed in their whole number form.
Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lbs 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, \$0.25 will not be accepted.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.