2015–2016 MATHCOUNTS® School Handbook

Contains 250 creative math problems
that meet the NCTM Grades 6-8 Standards.

Raytheon

2016 MATHCOUNTS
National Competition Sponsor

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Change the Equation has recognized MATHCOUNTS as having one of the nation's most effective STEM learning programs, listing the Math Video Challenge as an Accomplished Program in STEMworks.

APPROVED FOR LISTING NASSP 2015–2016 STUDENT CONTESTS & ACTIVITIES

The National Association of Secondary School Principals has placed all three MATHCOUNTS programs on the NASSP Advisory List of National Contests and Activities for 2015-2016.

HOW TO USE THIS SCHOOL HANDBOOK

If You're a New Coach

Welcome! We're so glad you're a coach this year.
Check out the **Guide for New Coaches**
starting on the next page.

If You're a Returning Coach

Welcome back! Thank you for coaching again.
Get the **2015-16 Handbook Materials**
starting on page 8.

Guide for New Coaches

Welcome to the MATHCOUNTS® Competition Series! Thank you so much for serving as a coach this year. Your work truly does make a difference in the lives of the students you mentor. We've created this Guide for New Coaches to help you get acquainted with the Competition Series and understand your role as a coach in this program.

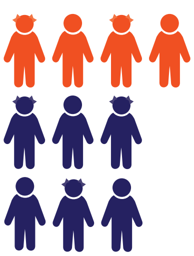
If you have questions at any point during the program year, please feel free to contact the MATHCOUNTS national office at (703) 299-9006 or info@mathcounts.org.

The MATHCOUNTS Competition Series in a Nutshell

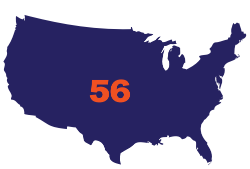
The **MATHCOUNTS Competition Series** is a national program that provides students the opportunity to compete in live, in-person math contests against and alongside their peers. Created in 1983, it is the longest-running MATHCOUNTS program and is open to all sixth-, seventh- and eighth-grade students.

HOW DOES IT WORK? The Competition Series has 4 levels of competition—school, chapter, state and national. Here's what a typical program year looks like.

Schools register in the fall and work with students during the year. Coaches administer the School Competition, usually in January. Any number of students from your school can participate in your team meetings and compete in the School Competition. MATHCOUNTS provides the School Competition to coaches in November. Many coaches use this to determine which student(s) will advance to the Chapter Competition.



Between 1 and 10 students from each school advance to the local Chapter Competition, which takes place in February. Each school can send a team of 4 students plus up to 6 individual competitors. All chapter competitors—whether they are team members or individuals—participate in the individual rounds of the competition; then just the 4 team members participate in the team round. Schools also can opt to send just a few individual competitors, rather than forming a full team. Over 500 Chapter Competitions take place across the country.



Top students from each Chapter Competition advance to their State Competition, which takes place in March. Your school's registration fees cover your students as far as they get in the Competition Series. If your students make it to one of the 56 State Competitions, no additional fees are required.

Top 4 individual competitors from each State Competition receive an all-expenses-paid trip to the National Competition, which takes place in May. These 224 students combine to form 4-person state teams, while also competing individually for the title of National Champion.

WHAT DOES THE TEST LOOK LIKE? Every MATHCOUNTS competition consists of 4 rounds—Sprint, Target, Team and Countdown Round. Altogether the rounds are designed to take about 3 hours to complete. Here's what each round looks like.

Sprint Round
40 minutes
30 problems total
no calculators used
focus on speed and accuracy

Target Round
Approx. 30 minutes
8 problems total
calculators used
focus on problem-solving and mathematical reasoning

The problems are given to students in 4 pairs. Students have 6 minutes to complete each pair.

Team Round
20 minutes
10 problems total
calculators used
focus on problem-solving and collaboration

Only the 4 students on a school's team can take this round officially.

Countdown Round
Maximum of 45 seconds per problem
no calculators used
focus on speed and accuracy

Students with highest scores on Sprint and Target Rounds compete head-to-head. This round is optional at the school, chapter and state level.

HOW DO I GET MY STUDENTS READY FOR THESE COMPETITIONS? What specifically you do to prepare your students will depend on your schedule as well as your students' schedules and needs. But in general, working through lots of different MATHCOUNTS problems and completing practice competitions is the best way to prepare to compete. Each year MATHCOUNTS provides the *School Handbook* to all coaches, plus lots of additional free resources online.

The next section of this Guide for New Coaches will explain the layout of the *MATHCOUNTS School Handbook* and other resources, plus give you tips on structuring your team meetings and preparation schedule.

The Role of the Competition Coach

Your role as the coach is such an important one, but that doesn't mean you need to know everything, be a math expert or treat coaching like a full-time job. Every MATHCOUNTS coach has a different coaching style and you'll find the style that works best for you and your students. But in general **every good MATHCOUNTS coach must do the following**.

- Schedule and run an adequate number of practices for participating students.
- Help motivate and encourage students throughout the program year.
- Select the 1-10 student(s) who will represent the school at the Chapter Competition in February.
- Take students to the Chapter Competition or make arrangements with parents and volunteers to get them there.

Looking for tools to help you become a top-notch coach? Check out Competition Math—a course for coaches from MATHCOUNTS + University of Oklahoma—on pg. 9!

You don't need to know how to solve every MATHCOUNTS problem to be an effective coach. In fact, many coaches have told us that they themselves improved in mathematics through coaching. Chances are, you'll learn with and alongside your students throughout the program year.

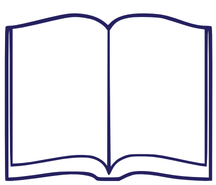
You don't need to spend your own money to be an effective coach. You can prepare your students using solely the free resources and this handbook. And, if you find that you want to do more with your students and need additional funding, you and your math team can participate in **Solve-A-Thon** to raise money for your MATHCOUNTS program. Learn more about this fundraiser at solveathon.mathcounts.org.

Check out Solve-A-Thon at solveathon.mathcounts.org to learn a free and easy way to raise extra funds for your math team.

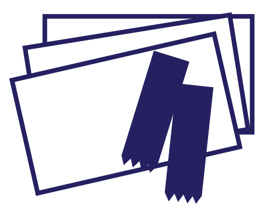
We give coaches numerous detailed resources and recognition materials so you can guide your Mathletes® to success even if you're new to teaching, coaching or competition math, and even if you use only the free resources MATHCOUNTS provides all competition coaches.

Making the Most of Your Resources

As the coach of a registered competition school, you already have received what we at MATHCOUNTS call the **School Competition Kit**. Your kit includes the following materials for coaches.



2015-2016 MATHCOUNTS School Handbook
The most important resource included in the School Competition Kit. Includes 250 problems.



Student Recognition Ribbons and Certificates
10 participation certificates and 1 ribbon for each registered chapter competitor.

You'll also get access to electronic resources. The following resources are available to competition coaches:

Official 2016 MATHCOUNTS School Competition
Released in November 2015 at www.mathcounts.org/coaches
Includes all 4 test rounds and the answer key

2015 MATHCOUNTS School, Chapter + State Competitions
Released in mid-April 2015 at www.mathcounts.org/pastcompetitions
Each level includes all 4 test rounds and the answer key

MATHCOUNTS Problem of the Week
Released each Monday at www.mathcounts.org/potw
Each multi-step problem relates to a timely event

You can use the **2016 MATHCOUNTS School Competition** to choose the students who will represent your school at the Chapter Competition. Sometimes coaches already know which students will attend the Chapter Competition. If you do not need the School Competition to determine your chapter competitors, then we recommend using it as an additional practice resource for your students.

The **2015-2016 MATHCOUNTS School Handbook** will be your primary resource for the Competition Series this year. It is designed to help your students prepare for each of the 4 rounds of the test, plus build critical thinking and problem-solving skills. This section of the Guide for New Coaches will focus on how to use this resource effectively for your team.

WHAT'S IN THE HANDBOOK? There is a lot included in the *School Handbook*, and you can find a full table of contents on pg. 8 of this book, but below are the sections that you'll use the most when coaching your students.

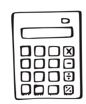
- **Handbook Problems:** 250 math problems divided into Warm-Ups, Workouts and Stretches. These problems increase in difficulty as the students progress through the book.
- **Solutions to Handbook Problems:** complete step-by-step explanations for how each problem can be solved. These detailed explanations are only available to registered coaches.
- **Answers to Handbook Problems:** key available to the general public. Your students can access this key, but not the full solutions to the problems.
- **Problem Index + Common Core State Standards Mapping:** catalog of all handbook problems organized by topic, difficulty rating and mapping to Common Core State Standards.

There are 3 types of handbook problems to prepare students for each of the rounds of the competition. You'll want to have your students practice all of these types of problems.

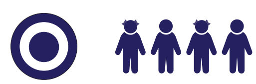
Warm-Ups
14 Warm-Ups in handbook
10 questions per Warm-Up
no calculators used

Warm-Ups prepare students particularly for the Sprint and Countdown Rounds.

Workouts
8 Workouts in handbook
10 questions per Workout
calculators used

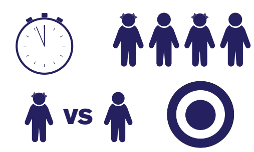


Workouts prepare students particularly for the Target and Team Rounds.



Stretches
3 Stretches in handbook
Number of questions and use of calculators vary by Stretch

Each Stretch covers a particular math topic that could be covered in any round. These help prepare students for all 4 rounds.



IS THERE A SCHEDULE I SHOULD FOLLOW FOR THE YEAR? On average coaches meet with their students for an hour once a week at the beginning of the year, and more often as the competitions approach. Practice sessions may be held before school, during lunch, after school, on weekends or at other times, coordinating with your school's schedule and avoiding conflicts with other activities.

Designing a schedule for your practices will help ensure you're able to cover more problems and prepare your students for competitions. We've designed the *School Handbook* with this in mind. Below is a suggested schedule for the program year that mixes in Warm-Ups, Workouts and Stretches from the *School Handbook*, plus free practice competitions from last year. This schedule allows your students to tackle more difficult problems as the School and Chapter Competition approach.

<table>
<tr><td>Mid-August – September 2015
Warm-Ups 1, 2 + 3
Workouts 1 + 2</td><td>October 2015
Warm-Ups 4, 5 + 6
Workout 3
Counting Stretch</td><td>November 2015
Warm-Ups 7 + 8
Workouts 4 + 5
Area Stretch</td><td>December 2015
Warm-Ups 9, 10 + 11
Workout 6
Modular Arithmetic Stretch</td></tr>
<tr><td colspan="2">January 2016
Warm-Ups 12, 13 + 14
Workouts 7 + 8
2016 MATHCOUNTS School Competition
Select chapter competitors (optional at this time)</td><td colspan="2">February 2016
Practice Competition: 2015 School Competition
Practice Competition: 2015 Chapter Competition
Select chapter competitors (required by this time)
2016 MATHCOUNTS Chapter Competition</td></tr>
</table>

You'll notice that in January or February you'll need to select the 1-10 student(s) who will represent your school at the Chapter Competition. This must be done before the start of your local Chapter Competition. You'll submit the names of your chapter competitors either online at www.mathcounts.org/coaches or directly to your local Chapter Coordinator.

It's possible you and your students will meet more frequently than once a week and need additional resources. If that happens, don't worry! You and your Mathletes can work together using the **Interactive MATHCOUNTS Platform**, powered by NextThought. This free online platform contains numerous *MATHCOUNTS School Handbooks* and past competitions, not to mention lots of features that make it easy for students to collaborate with each other and track their progress. Learn more on pgs. 12-13 of this handbook.

Use the free Interactive MATHCOUNTS Platform at mathcounts.nextthought.com to get even more handbook problems and past competitions!

And remember, just because you and your students will meet once a week doesn't mean your students can only prepare for MATHCOUNTS one day per week. Many coaches assign "homework" during the week so they can keep their students engaged in problem solving outside of team practices. Here's one example of what a 2-week span of practices in the middle of the program year could look like.

Monday	Tuesday	Wednesday (Weekly Team Practice)	Thursday	Friday
-Students continue to work on Workout 4, due Wednesday	-Students continue to work on Workout 4 -Coach emails team to assign new Problem of the Week, due Wednesday	-Coach reviews solutions to Workout 4 -Coach gives Warm-Up 7 to students as timed practice and then reviews solutions -Students discuss solutions to Problem of the Week in groups	-Coach emails math team to assign Workout 5 as individual work, due Wednesday	-Students continue to work individually on Workout 5
-Students continue to work individually on Workout 5, due Wednesday	-Students continue to work on Workout 5 -Coach emails team to assign new Problem of the Week, due Wednesday	-Coach reviews solutions to Workout 5 -Coach gives Warm-Up 8 to students as timed practice and then reviews solutions -Students discuss solutions to Problem of the Week in groups	-Coach emails math team to assign Workout 6 as group work, due Wednesday	-Students work together on Workout 6 using online Interactive Platform

WHAT SHOULD MY TEAM PRACTICES LOOK LIKE? Obviously every school, coach and group of students is different, and after a few practices you'll likely find out what works and what doesn't for your students. Here are some suggestions from veteran coaches about what makes for a productive practice.

- Encourage discussion of the problems so that students learn from each other
- Encourage a variety of methods for solving problems
- Have students write math problems for each other to solve
- Use the **Problem of the Week** (posted to www.mathcounts.org/potw every Monday)
- Practice working in groups to develop teamwork (and to prepare for the Team Round)
- Practice oral presentations to reinforce understanding

Below is a sample agenda for a 1-hour practice session. There are many ways you can structure math team meetings and you will likely come up with an agenda that works better for you and your group. It also is probably a good idea to vary the structure of your meetings as the program year progresses.

MATHCOUNTS Team Practice Sample Agenda – 1 Hour

Review Problem of the Week (20 minutes)

- Have 1 student come to the board to show how s/he solved the first part of the problem.
- Discuss as a group other strategies to solve the problem (and help if student answers incorrectly).
- Have students divide into groups of 4 to discuss the solutions to the remaining parts of the problem.
- Have 2 groups share answers and explain their solutions.

Timed Practice with Warm-Up (15 minutes)

- Have students put away all calculators and have one student pass out Warm-Ups (face-down).
- Give students 12 minutes to complete as much of the Warm-Up as they can.
- After 12 minutes is up, have students hold up pencils and stop working.

Play Game to Review Warm-Up Answers (25 minutes)

- Have students divide into 5 groups (size will depend on number of students in meeting).
- Choose a group at random to start and then rotate clockwise to give each group a turn to answer a question. When it is a group's turn, ask the group one question from the Warm-Up.
- Have the group members consult their completed Warm-Ups and work with each other for a maximum of 45 seconds to choose the group's official answer.
- Award 2 points for a correct answer on questions 1-3, 3 points for questions 4-7 and 5 points for questions 8-10. The group gets 0 points if they answer incorrectly or do not answer in 45 seconds.
- Have all students check their Warm-Up answers as they play.
- Go over solutions to select Warm-Up problems that many students on the team got wrong.

Register for the National Math Club to get fun math games and activities! Go to www.mathcounts.org/club or pg. 14 for more information.

OK I'M READY TO START. HOW DO I GET STUDENTS TO JOIN? Here are some tips given to us from successful competition coaches and club leaders for getting students involved in the program at the beginning of the year.

- Ask Mathletes who have participated in the past to talk to other students about participating.
- Ask teachers, parent volunteers and counselors to help you recruit.
- Reach parents through school newsletters, PTA meetings or Back-to-School-Night presentations.
- Advertise around your school by:
    1. posting intriguing math questions (specific to your school) and referring students to the first meeting for answers.
    2. designing a bulletin board or display case with your MATHCOUNTS poster and/or photos and awards from past years.
    3. attending meetings of other extracurricular clubs (such as honor society) so you can invite their members to participate.
    4. adding information about the MATHCOUNTS team to your school's website.
    5. making a presentation at the first pep rally or student assembly.

Good luck in the competition! If you have any questions during the year, please contact the MATHCOUNTS national office at (703) 299-9006 or info@mathcounts.org.

2015-16 Handbook Materials

Thank you for being a coach in the MATHCOUNTS Competition Series this year!
We hope participating in the program is meaningful and enriching for you and your Mathletes.

What's in This Year's Handbook

What's New This Year

We're always trying to find ways to make MATHCOUNTS programs even better for coaches and students. Check out our new tools and resources designed exclusively for coaches and Mathletes!

Competition Math Online Course for Coaches

Opening this fall, **Competition Math for Middle School** is a new online course designed to provide math competition coaches valuable strategies, tools and resources for training their students. Whether you're a first-time coach or a returning coach looking for new strategies to engage and train your students, this course will help you take your coaching to the next level.

- Exclusive content created by **Art of Problem Solving**.
- Expert advice from **MATHCOUNTS** content creators and Art of Problem Solving instructors on how to explain the most common types of math competition problems.
- Tips from national-level MATHCOUNTS coaches on how to keep students motivated and engaged throughout the year.
- Opportunities to engage with classmates.

This course will take place October 5 – December 11, 2015 through the **University of Oklahoma** on the NextThought/Janux online platform. All coaches who successfully complete the course will receive one graduate credit from the University of Oklahoma, which most educators can use for continuing education credits at their district.

Registration is open now through October 5th! Enrollment in this course costs $199. Learn more and enroll at mathcounts.ou.edu.

MATHCOUNTS Trainer App

Help your Mathletes train their way to Nationals with the free **MATHCOUNTS Trainer**, presented by **Art of Problem Solving**, available at www.aops.com/mathcounts_trainer now and coming as an app this fall!

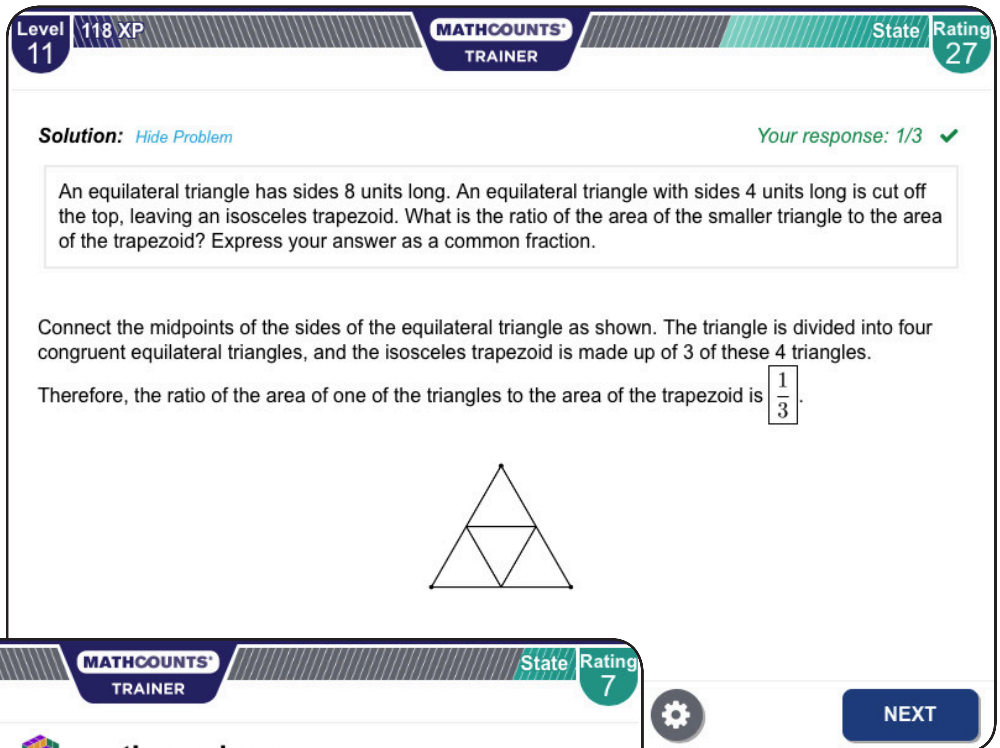
MATHCOUNTS® TRAINER

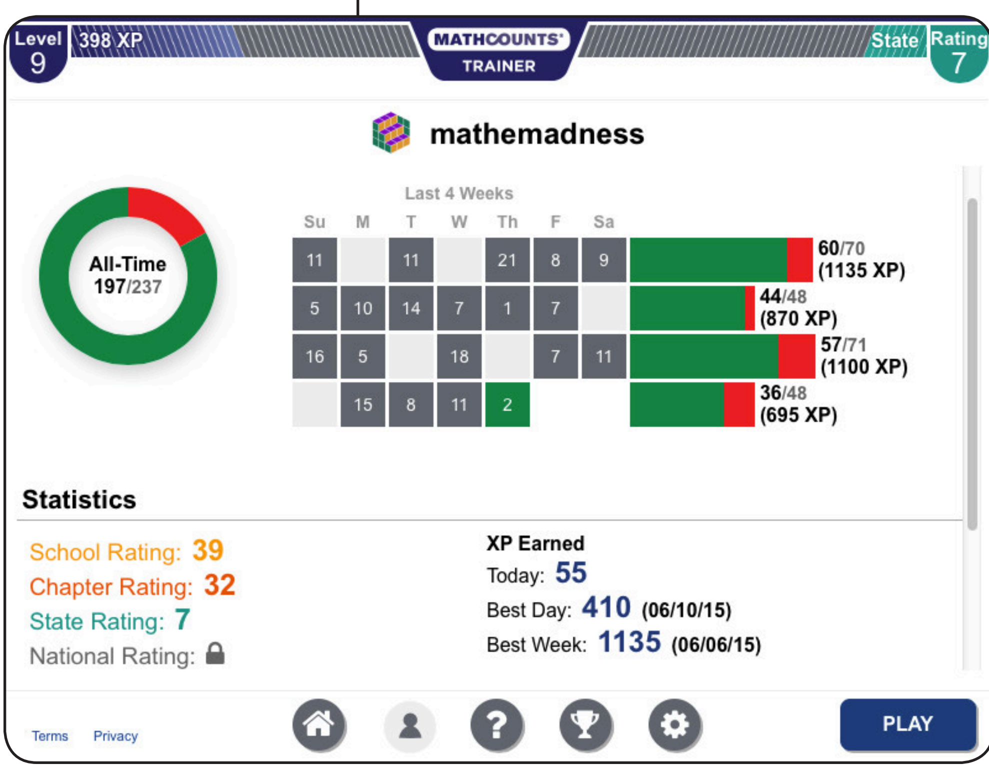
By AoPS.com Art of Problem Solving

With thousands of problems from previous MATHCOUNTS School, Chapter, State, and National Competitions, the Trainer is great practice for aspiring Mathletes of all levels.

- *Full solutions* are provided for every problem so players can learn how to approach even the toughest questions.
- *Real-time dashboards* allow players to track their progress and see their standing compared to other players.

Encourage your Mathletes to compete against others and see if they can climb to the top of the Leaderboard!





Critical 2015-2016 Dates

2015

Aug. 17 – Dec. 11	Submit your school's registration to participate in the Competition Series and receive this year's School Competition Kit, which includes a hard copy of the *2015-2016 MATHCOUNTS School Handbook*. Kits are shipped on an ongoing basis between mid-August and December 31. The fastest way to register or add more students to your Competition Series registration is online at **www.mathcounts.org/compreg**. You also can mail, email or fax the MATHCOUNTS Competition Series Registration Form with payment to: MATHCOUNTS Foundation – Competition Series Registrations 1420 King Street, Alexandria, VA 22314 *Email*: reg@mathcounts.org *Fax*: (703) 299-5009 Questions? Call the MATHCOUNTS national office at (703) 299-9006.
Nov. 2	The 2016 School Competition will be available online. All registered coaches can log in at www.mathcounts.org/coaches to download the competition.
Nov. 13 (postmark)	**Deadline to register for the Competition Series at reduced registration rates** ($90 for a team and $25 for each individual). After November 13, registration rates will be $100 for a team and $30 for each individual.
Dec. 11 (postmark)	**Competition Series Registration Deadline** In some circumstances, late registrations might be accepted at the discretion of MATHCOUNTS and the local coordinator. *Late fees will apply. Register on-time to ensure your students' participation.*

2016

Early Jan.	If you have not been contacted with details about your upcoming competition, call your local or state coordinator. Coordinator contact information is available at www.mathcounts.org/findmycoordinator.
Late Jan.	If you have not received your School Competition Kit, contact the MATHCOUNTS national office at (703) 299-9006 or info@mathcounts.org.
Jan. 30 – Feb. 29	**Chapter Competitions**
March 1-31	**State Competitions**
May 8-9	**2016 Raytheon MATHCOUNTS National Competition in Washington, DC**

Helpful Resources

MATHCOUNTS Solve-A-Thon

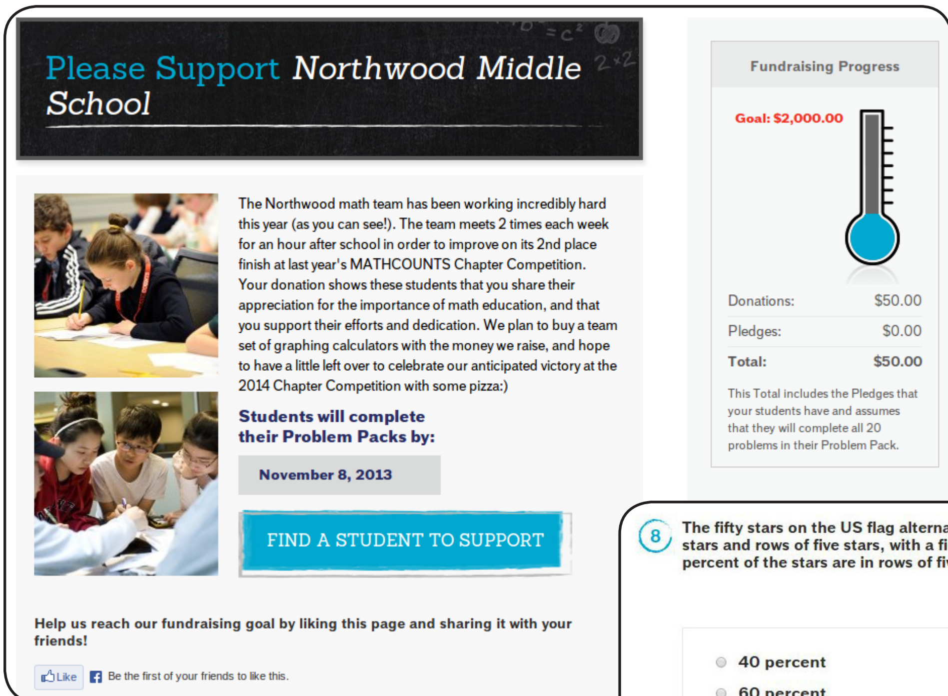
Solve-A-Thon is a free fundraiser that empowers students and teachers to use math to raise money for the math programs at their school. All money raised goes to support math programming that benefits the students' local communities, with 60% going directly back to the school. Launched last year, Solve-A-Thon was designed with teachers in mind; participating is free, and getting started takes just a couple of minutes. Here's how the fundraiser works:

1. Teachers and students sign up and **create** online fundraising pages that explain why they value math and why they are raising money for their school's math program.

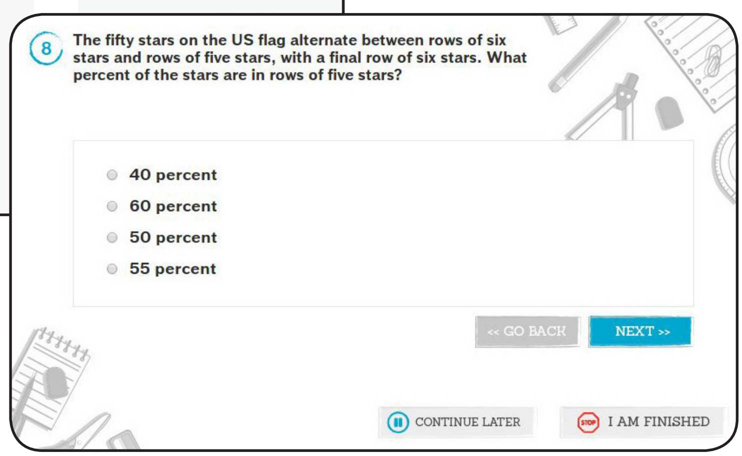
2. Students **share** the links to their fundraising pages with friends, family and local community members to earn donations and pledges.

3. Students **solve** an online Problem Pack with 20 math problems covering topics from the NCTM Grades 6-8 Standards.

4. Students, teachers, schools and local communities **win** critical funding, prizes and improved math programs.



Unlike traditional fundraisers, there is no door-to-door selling and no tracking of sales or money raised. Students participate in a meaningful learning activity to raise money, and money earned is tracked online automatically. Learn more and sign up at solveathon.mathcounts.org.

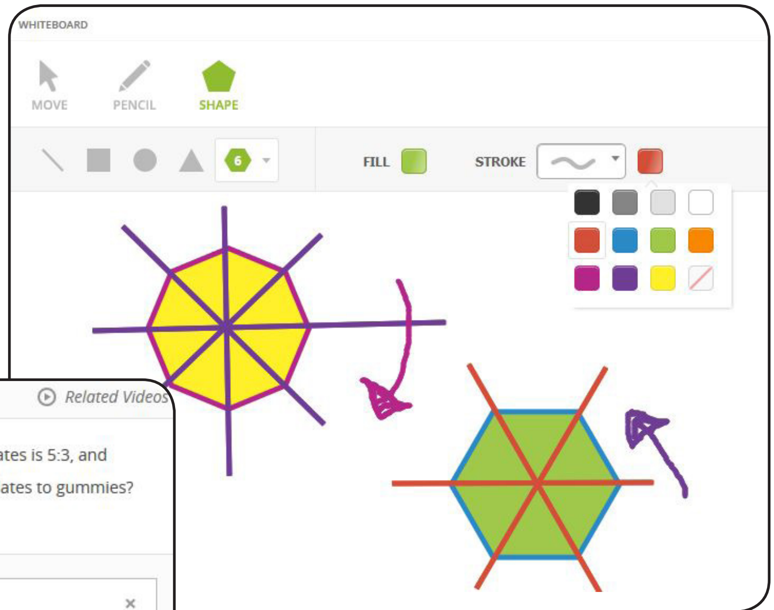


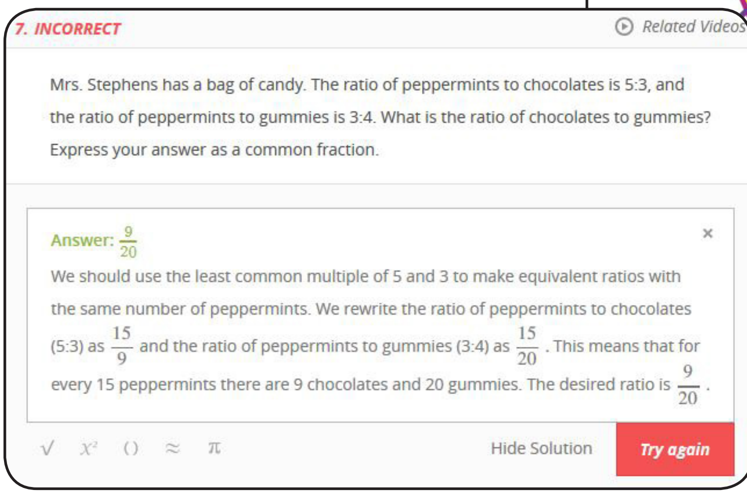
Interactive MATHCOUNTS Platform

The **Interactive MATHCOUNTS Platform** provides a unique forum where members of the MATHCOUNTS community can collaborate, chat and utilize innovative online features as they work on problems from MATHCOUNTS handbooks and competitions.

Powered by **NextThought**, the Interactive MATHCOUNTS Platform continues to grow, with more problems and features being added every year. This resource includes problems from the *2015-2016 MATHCOUNTS School Handbook*, as well as past handbooks and School, Chapter and State Competitions from multiple years. Users can take advantage of numerous features that make this platform engaging; here are just a few:

- *Digital white-boards* enable students to highlight problems, add notes and questions and show their work.
- *Interactive problems* can be used to assess student or team performance.
- *Advanced search tools* make it easy to find MATHCOUNTS content and notes.
- *Collaborative forums* allow users to chat and share with the global MATHCOUNTS community.





The Interactive MATHCOUNTS Platform is a great addition to your competition preparations and team practices because it allows your Mathletes to work together on problem sets in a fun way. Create your free account at mathcounts.nextthought.com today!

MATHCOUNTS OPLET

The **Online Problem Library and Extraction Tool** is an online database of over 13,000 problems and over 5,000 step-by-step solutions. OPLET subscribers can create personalized worksheets/quizzes, flash cards and Problems of the Day with 15 years worth of MATHCOUNTS handbook and competition problems.

With OPLET, creating original resources is easy. You can personalize the materials you create in the following ways:

- *Format:* worksheet/quiz, flash cards or Problem of the Day
- *Range of years of MATHCOUNTS materials*
- *Difficulty level:* 5 levels from easy to difficult
- *Number of questions*
- *Solutions included/omitted for select problems*
- *MATHCOUNTS usage:* filters by competition round or handbook problem type
- *Math concept:* including arithmetic, algebra, geometry, counting/probability, number theory

A 12-month **OPLET subscription** costs $275, and schools registering students in the Competition Series receive a $5 discount per registered student (up to $50 off). Plus, if you purchase OPLET by October 16, 2015, you can save an additional $25, for a total savings of up to $75 (refer to coupon at right). Learn more and purchase your subscription at www.mathcounts.org/oplet.

$25 OFF

when you buy or renew your OPLET subscription by October 16, 2015

Renewing Subscribers:
use coupon code RENEW1516

New Subscribers:
use coupon code OPLET1516

Other MATHCOUNTS Programs

MATHCOUNTS was founded in 1983 as a way to provide new avenues of engagement in math for middle school students. MATHCOUNTS began solely as a competition, but has grown to include 3 unique but complementary programs: the **MATHCOUNTS Competition Series**, the **National Math Club** and the **Math Video Challenge**.

Your school can participate in all 3 MATHCOUNTS programs! The M*ATHCOUNTS School Handbook* is primarily a competition resource, but it can be used as a resource when participating in the other programs, as well. Below is some additional information about the National Math Club and the Math Video Challenge.

THE NATIONAL MATHCLUB powered by MATHCOUNTS®

The **National Math Club** is a free enrichment program that provides teachers and club leaders with resources to run a math club. The materials provided through the National Math Club are designed to engage students of all ability levels—not just the top students—and are a great supplement for classroom teaching. This program emphasizes collaboration and provides students with an enjoyable, pressure-free atmosphere in which they can learn math at their own pace.

Active clubs also can earn rewards by having a minimum number of club members participate (based on school/organization/group size). There is no cost to sign up for the National Math Club, and registration is open to schools, organizations and groups that consist of at least 4 students in 6th, 7th and/or 8th grade and have regular in-person meetings. More information can be found at www.mathcounts.org/club, and the 2015-2016 School Registration Form is included on the next page.

The MATHCOUNTS School Handbook *is supplemental to the National Math Club. Resources in the* Club Activity Book *will be better suited for more collaborative and activity-based club meetings.*

MATHVIDEO CHALLENGE produced by MATHCOUNTS®

The **Math Video Challenge** is an innovative program that challenges students to work in teams of 4 to create a video explaining the solution to a MATHCOUNTS problem and demonstrating its real-world application. This project-based activity builds math, communication and collaboration skills.

Students post their videos to the contest website, where the general public votes for the best videos. The 100 videos with the most votes advance to judging rounds, in which 20 semifinalists and, later, 4 finalists are selected. This year's finalists will present their videos to the students competing at the 2016 Raytheon MATHCOUNTS National Competition, and the 224 Mathletes will vote to determine the winner. Members of the winning team receive college scholarships. Registration is open to all 6th, 7th and 8th grade students. More information can be found at videochallenge.mathcounts.org.

Students must base their video for the 2015-2016 Math Video Challenge on a problem they select from the 2015-2016 MATHCOUNTS School Handbook.

The fastest way to register your school is online at **www.mathcounts.org/clubreg**!

THE NATIONAL MATHCLUB
powered by MATHCOUNTS®

2015-2016 **SCHOOL** REGISTRATION FORM

This registration form is for U.S. middle schools only. To register a non-school group (such as a Girl Scout Troop, Boys and Girls Club Chapter or math circle) for the National Math Club, please go to www.mathcounts.org/club to review eligibility requirements and register.

Step 1: Fill in your school's name and confirm eligibility to participate.

*required information

☒ ***U.S. school with students in 6th, 7th and/or 8th grade***

*School Name** ______________________________

***There can be multiple clubs at the same U.S. school**, as long as each club has a different club leader.*

By signing below I, the club leader, affirm that the school named above is a U.S. school with students in sixth-, seventh- and/or eighth-grade and is therefore eligible to participate in the National Math Club. I affirm that I have permission to register the school above for this program and I understand that MATHCOUNTS can cancel my membership at any time if it is determined that my group is ineligible.

Club Leader Signature* ______________________________

Step 2: Provide your information so we can send you materials and set up your online access.

*required information

Club Leader Name* ______________________________ Club Leader Phone ______________

Club Leader Email Address* ______________________________

Club Leader Alternate Email Address ______________________________

Club Address* ______________________________

City, State ZIP* ______________________________

Estimated total number of participating students in club (minimum 4 students)*: __________

☐ My school previously participated in the National Math Club.

School Type (please check one)*: ☐ Public ☐ Charter ☐ Private ☐ Homeschool ☐ Virtual

Department of Defense or State Department schools, please provide additional information below.

Clubs located outside of the U.S. states or territories are not eligible to participate in the National Math Club unless they are in schools affiliated with the U.S. Department of Defense or State Department.

My school is sponsored by (please check one): ☐ U.S. Department of Defense (DoDDS)
☐ U.S. State Department

Country ______________________________

Step 3: Almost done... just turn in your form.

Mail, email a scanned copy or fax this completed form to:
MATHCOUNTS Foundation, 1420 King Street, Alexandria, VA 22314
Email: reg@mathcounts.org
Fax: 703-299-5009

Questions?
Please call the national office at
703-299-9006

Warm-Up 1

1. ____________ If Franco types *MATHCOUNTS* 50 times in total, how many more consonants does he type than vowels?

2. ________ seconds Vicki and Candace start a race at the same time. Vicki finishes in 28 minutes 47 seconds. Candace finishes in 29 minutes 46 seconds. How many seconds ahead of Candace does Vicki finish?

3. ________ degrees What is the sum of the degree measures of the interior angles of a regular octagon?

4. $ ____________ Vera's favorite coffee blend costs $1.32 per ounce, including tax. How much will it cost Jorge to buy Vera one and a half pounds of her favorite coffee?

5. ________ ways Lily is going to the movies with Abby, Bea and Jaclyn. Abby wants to sit at the end of a row, and Bea only cares that she is seated next to Jaclyn. In how many different ways can the girls be seated in a single row that has only four seats?

6. ____________ On the first five math quizzes of the school year, Maurice's scores were 80, 84, 99, 92 and 100. After the sixth quiz, Maurice's median score for all six quizzes was 90. What was Maurice's score on the sixth quiz?

7. ________ regions What is the maximum number of non-overlapping regions that can be determined by three lines in a plane?

8. ____________ If $3x + 155 = 272$, then what is the value of $3x + 160$?

9. ____________ Ash's secret number is a factor of 72. The secret number is neither prime nor a multiple of 3. What is the sum of all possible values of Ash's secret number?

10. ____________ One-half of three-fourths of a number is 20 more than two-fifths of the same number. What is the number?

Warm-Up 2

11. ________ What is the least possible sum of the digits displaying the time on a 12-hour digital clock?

12. ________ palin-dromes How many 4-digit palindromes contain both the digits 1 and 2?

13. ________ cm What is the radius of a circle with a circumference measuring 24π cm?

14. ________ On a number line, what number is two-thirds of the distance from one-half to 2.25? Express your answer as a common fraction.

15. ________ m^2 The length of a rectangular sports field is three times its width. If the perimeter of the field is 880 meters, what is the area of the field?

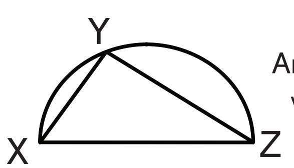
16. ________ An item with an original price of *d* dollars has its price increased by *x* percent in April and then decreased by *x* percent in May. The resulting price is 4 percent less than the original price. What is the value of *x*?

17. ________ rows Letter *M*s are stacked, as shown, so that the top row has one *M*, the second row has two *M*s, the third row has three *M*s, and so on. What is the least number of rows required for the total number of stacked *M*s to be divisible by 7?

M
M M
M M M
M M M M

18. ________ minutes Josh mowed one-third of a 2000-ft^2 lawn in 18 minutes. At the same rate, how many minutes would it take him to mow a 4200-ft^2 lawn? Express your answer to the nearest whole number.

19. ________ $units^2$



Arc XYZ, shown here, is a semicircle. If XZ = 15 units, what is the value of $(XY)^2 + (YZ)^2$?

20. ________ Seven contestants enter a drawing that begins with 100 balls numbered 1 through 100 in a box. Each contestant randomly selects a ball without replacement. The two contestants who select balls with the two highest numbers each will win a cash prize. The first six contestants select balls numbered 83, 5, 44, 67, 21 and 30. What is the probability that the last contestant will win a cash prize? Express your answer as a common fraction.

Warm-Up 3

21. ________ shifts A shift consists of rotating a two-digit number 90 degrees clockwise and then exchanging the position of the two digits. How many shifts must be performed before the number 19 returns to its original orientation?

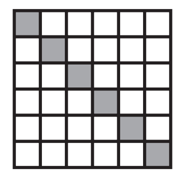
22. ________ people The Population Reference Bureau reported a net gain of 155 people on Earth each minute. At this rate, how many more people are there on Earth every day?

23. ________ If $\frac{1}{x} + \frac{1}{x} + \frac{1}{x} + \frac{1}{x} = 24$, what is the value of x? Express your answer as a common fraction.

24. ________ times Maximo wrote down two rows of numbers as shown. In the first row, he wrote the positive multiples of 7, starting with 7 and ending with 700. In the second row, he wrote the positive multiples of 13, starting with 13 and ending with 1300. How many times did corresponding pairs of terms in the first and second rows have the same units digit?

First row:	7	14	21	28	35	…	700
Second row:	13	26	39	52	65	…	1300

25. ________ In the 6 × 6 array of squares shown, two squares are adjacent if they share a side. What is the probability that two adjacent squares, chosen at random, are not the same color? Express your answer as a common fraction.



26. ________ integers A *very round number* is a positive integer that has exactly one nonzero digit. How many integers less than one billion are very round numbers?

27. ________ Adding $\frac{3}{8}$ to a number increases it by 5% of its original value. What is the original number? Express your answer as a decimal to the nearest tenth.

28. ________ cm What is the height of a right rectangular prism with a length of 4 cm, a width of 3 cm and a volume of 108 cm^3?

29. ________ minutes Ms. Swift wrote 40 pages of notes, each page containing approximately 240 words. Chanel, who types 36 words per minute, volunteered to type some of Ms. Swift's notes, beginning at the first page and progressing through them in order. Marcus, who types 54 words per minute, volunteered to type some of Ms. Swift's notes, beginning at the last page and progressing through them in reverse order. If Chanel and Marcus begin at the same time, working together, how many minutes will it take them to finish typing the notes? Express your answer to the nearest whole number.

30. ________ days An exponentially talented salesman sells 1 car on his first day, 2 cars on his second day, 4 cars on his third day and so on, so that every day after the first, he sells twice as many cars as the day before. How many days does it take him to sell a total of at least 1000 cars?

Workout 1

31. ________ m On average, a human fingernail grows at a rate of 3 mm per month. At this rate, how many total meters will Deshawn's 10 fingernails grow over the next 10 years? Express your answer as a decimal to the nearest tenth.

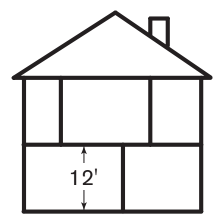
32. ________ % What percent of the prime numbers less than 100 have a units digit of 3?

33. ________ inches In Mrs. Abel's class, there are 16 girls and 14 boys. The mean of the girls' heights is 63.7 inches, and the mean of the boys' heights is 65.4 inches. What is the mean of the heights of all 30 students? Express your answer as a decimal to the nearest tenth.

34. $ ________ The price P, in dollars, that AppCo charges annual subscribers for downloading x smartphone applications is calculated using the formula $P = 5 + 1.25x$. If a subscriber downloaded 25 applications during a year, what was the average cost per application?

35. ________ members The local gaming club has more than 2 but fewer than 20 members. Every member of the group purchased a signed copy of the latest book from the legendary Dumas, for a grand total of exactly $299. If each member paid the same whole number dollar amount, how many members are in the club?

36. ________ inches Jamee is building a two-story house in which the top floor is 12 feet above the bottom floor. The local building code specifies that the height of a step cannot exceed 7.5 inches. What is the maximum height of a step in a staircase that Jamee can construct between the two floors if all steps will have the same height? Express your answer as a decimal to the nearest tenth.



37. ________ mm The first and second books of a book series have 10 and 11 chapters, respectively, and each chapter has exactly 20 pages. The total thickness of the pages in the first book is 14 mm. For book two, the total thickness of the pages plus the front and back covers is 17 mm. The thickness of each page in book one is the same as that in book two, and the thickness of each cover in book one is the same as that of book two. What is the total thickness when the two books are stacked cover to cover? Express your answer as a decimal to the nearest tenth.

38. ________ The line that contains (4, −7) and (−3, 14) also contains the point (0, b). What is the value of b?

39. ________ What is the greatest multiple of 10 that can be expressed using only the numbers 2, 2, 3 and 5, each exactly once, with any or all of the operations +, −, × and ÷, exponentiation and parentheses? Express your answer in scientific notation.

40. ________ cm Two sides of a triangle are 15 cm and 18 cm in length. The altitude to the 18-cm side is 10 cm. What is the length of the altitude to the 15-cm side?

Workout 2

41. ________ triangles Point A is a vertex of a regular octagon. When all possible diagonals are drawn from point A in the polygon, how many triangles are formed?

42. ________ in^2 Alicia is framing a rectangular photo that measures 8 inches high by 12 inches wide. However, the frame she has was designed to hold a picture with sides in the ratio 4:5. If the frame Alicia has is the smallest possible frame that holds the photo, how much extra area will there be? Express your answer as a decimal to the nearest tenth.

43. $ ________ If four hamburgers and two hot dogs cost $16.40, and six hamburgers and four hot dogs cost $26.40, what is the combined cost of a hamburger and a hot dog?

44. ________ % At Grace Hopper Middle School, there are 531 students in the sixth, seventh and eighth grades combined. The table below shows the number of students in each grade who play soccer and the number of students in each grade who do not play soccer. If all of the seventh and eighth graders attend an assembly, and two students at the assembly are chosen at random, what is the probability, as a percent, that neither student plays soccer? Express your answer to the nearest tenth.

Grade	Play Soccer	Don't Play Soccer
6	79	118
7	61	116
8	54	103

45. $ ________ It costs $0.12 for the Cheapco Soda Company to manufacture enough cola to fill a cylindrical container that is 2 inches in diameter and 4 inches in height. How much would it cost for Cheapco to manufacture enough cola to fill a cylindrical container that is 6 inches in diameter and 10 inches in height?

46. ________ minutes Kim created decorations for the school dance. It took her 4 minutes to create the first decoration, and each decoration after the first one took 10% less time to create than the one before it. How many minutes did it take her to create the first five decorations? Express your answer as a decimal to the nearest tenth.

47. ________ cm Lily made a circle from a length of string measuring 26 cm, and Willow made a circle from a length of string measuring 31 cm. What is the absolute difference in the radii of their circles? Express your answer as a decimal to the nearest tenth.

48. ________ The digits of the addends in the sum shown are represented by the letters A, B and G. What is the value of A × (B + G)?

$$\begin{array}{cccc} & G & A & B \\ + & B & A & G \\ \hline 1 & 0 & 9 & 0 \end{array}$$

49. ________ ways In how many ways can the integers 1 through 6 be written horizontally in a row so that the sum of any two adjacent integers is odd?

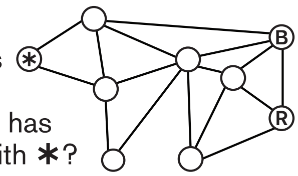
50. ________ units What is the radius of the largest sphere that will fit inside a cube of volume 8 $units^3$?

Warm-Up 4

51.__________ What is the smallest integer that is greater than the product 2.4999 × 3.9999 × 4.9999?

52.__________ m^2 What is the area of a rectangle with sides that measure $2\frac{1}{2}$ and $7\frac{1}{3}$, in meters? Express your answer as a mixed number.

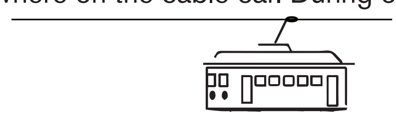
53.__________ In the figure, each circle is a vertex of one or more triangles. The circles marked B and R are colored blue and red, respectively. If each of the remaining circles is to be colored red, blue or yellow so that no triangle has two vertices of the same color, what is the color of the circle marked with ∗?



54.__________ The mean of the first five terms of an arithmetic sequence is 27, and the mean of the first eight terms of the same sequence is 39. What is the absolute difference between the first and second terms of this sequence?

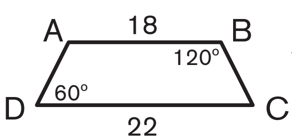
55.__________ people What is the minimum number of people that must be in a room to guarantee that at least one person has a birthday whose day is a single digit, assuming that no two people have the same birthday?

56.__________ feet A cable car is 30 feet in length and travels back and forth in a straight line, on a single cable that is 3000 feet in length. Let *P* be a point somewhere on the cable car. During one round trip, what is the distance traveled by point *P*?



57.__________ To cater a party, Fancy Caterers charges a certain amount per guest plus a fixed delivery fee. The total cost for a party with 25 guests is twice the cost of a party with 10 guests. If the cost of a party with *n* guests is twice the cost of a party with 25 guests, what is the value of *n*?

58.__________ cm



ABCD is a trapezoid with bases AB = 18 cm and DC = 22 cm. The measures of ∠ABC and ∠ADC are 120 degrees and 60 degrees, respectively. What is the perimeter of trapezoid ABCD?

59.__________ seconds Kaci's jogging speed is 60% faster than her walking speed. The time it takes her to walk a mile is $5\frac{1}{2}$ minutes longer than the time it takes her to jog a mile. How many seconds does it take Kaci to jog a mile?

60.__________ % Thirty percent of 80% of 20 is the same as what percent of 60% of 40?

Warm-Up 5

61. ______ coins Zoey has \$2.90 in quarters, dimes and nickels. The number of nickels is one more than six times the number of quarters, and the number of dimes is four times the number of quarters. What is the total number of coins Zoey has?

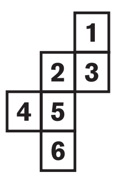
62. ______ days Every day, a cafeteria offers pizza, chicken and salad as lunch entrées. This comes with a choice of milk, juice, water or tea to drink and a cookie or brownie for dessert. If a different combination of one entrée, one drink and one dessert is tried every day, how many days does it take to try every combination?

63. ______ prisms A pile of clay can be molded to form a solid cube with edges of length 10 cm. How many solid rectangular prisms with dimensions 2 × 4 × 5 cm can be made from the same amount of clay?

64. \$ ______ A restaurant offers an incentive by giving a \$5 coupon for every \$40 spent during a calendar year. If Roberto spends \$639 at the restaurant in a year, what is the total dollar value of the coupons he can expect to receive?

65. ______ golf balls GG's shipping company ships golf balls in three different box sizes. The small box contains a dozen golf balls. The medium box is similar to the small box, but each dimension is doubled. The large box also is similar to the small box, but each dimension is tripled. If Jake orders one box of each size, how many golf balls will he receive?

66. ______



This figure, composed of six congruent squares labeled 1 through 6, is a net of a cube. If it is folded to form a cube, what is the sum of the digits on the four faces that are adjacent to the face labeled 1?

67. ______ Two distinct numbers are selected at random from the set {1, 2, 3, 4, 5, 6}. What is the probability that their product is an odd number? Express your answer as a common fraction.

68. ______ If today is Friday, what day of the week will it be 101 days from today?

69. base ______ Thomas learned to count in a base other than 10. Instead of writing 163, which is in base 10, Thomas writes 431. What base is Thomas using?

70. ______ units Starting at the origin of a coordinate plane, an ant crawls 1 unit to the right, 2 units up, 3 units to the right, 4 units up, 5 units to the right and 6 units up. How far from the origin is the ant currently located?

Warm-Up 6

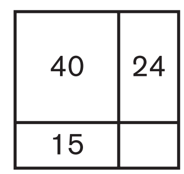
71. __________ When an integer is doubled and increased by 3, the result is 5 less than the square of the integer. What is the sum of all such integers?

72. __________ degrees Triangle ABC is isosceles with AB = AC and $m\angle A = 30$ degrees. Side AB is extended to D so that $m\angle ACD = 90$ degrees. What is the degree measure of $\angle BCD$?

73. __________ games If the15 teams in a soccer league each play eight games in a season, what is the total number of games played during the season?

74. __________ miles Laree notices that the current mileage on her car is a multiple of 1000 and is a perfect cube. She does some calculations and determines that she will have to drive another 4921 miles before the number of miles on her car is a perfect cube again. What is Laree's current mileage?

75. __________ units^2 Two lines parallel to the sides of a large rectangle divide the rectangle into four regions. The areas of three of the regions, starting in the upper right corner and going counterclockwise, are 24, 40 and 15 units^2 as shown (not to scale). What is the area of the large rectangle?



76. __________ The first three terms of an arithmetic sequence are $17x + 20$, $18x - 3$ and $20x + 1$, in that order. What is the value of x?

77. __________ mg The effectiveness of Donovan's cold medication decreases geometrically, retaining one-fourth of its original effectiveness after four hours. If Donovan takes 500 mg of medication every four hours beginning at 8:00 a.m., how much effective medication remains in his body at 6:00 p.m.? Express your answer as a mixed number.

78. __________ marbles Ron and Martin are playing a game with a bowl containing 39 marbles. Each player takes turns removing 1, 2, 3 or 4 marbles from the bowl. The person who removes the last marble loses. If Ron takes the first turn to start the game, how many marbles should he remove to guarantee he is the winner?

79. __________ What common fraction is equal to the sum $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{512}$?

80. $__________ Mrs. Lowe is buying lunch for her class of 25 students. A large pizza that serves three people costs $8, and a giant sub that serves four people costs $9. If pizzas and subs cannot be purchased in part, what is the least amount it will cost Mrs. Lowe to feed all the students in her class?

Workout 3

81. ________ terms How many terms of the sequence $3^1, 3^2, 3^3, \ldots, 3^{100}$ have a units digit of 7?

82. ________ miles A car service charges \$2.40 for the first quarter of a mile traveled and then charges \$0.40 for each additional fifth of a mile. How many miles can a rider travel for \$10? Express your answer as a decimal to the nearest hundredth.

83. ________ Billie is downloading a video game to her computer. When she checks at 11:15 a.m., her computer indicates that the download is 35% complete. When she checks again at 11:30 a.m., the download is 80% complete. Assuming the video game is downloaded at a constant rate, Billie should expect the download to be complete after an additional a minutes b seconds. If a and b are whole numbers and $b < 60$, what is the value of $a \times b$?

84. ________ % On average, one out of every 70 widgets manufactured has a defect. If Nick inspects four widgets chosen at random, what is the probability, expressed as a percent, that at least one of them will be defective? Express your answer to the nearest tenth.

85. ________ inches A newly minted half-dollar coin has a diameter of 1.205 inches and 150 equally spaced ridges around its circumference. What is the distance between any two adjacent ridges? Express your answer as a decimal to the nearest thousandth.

86. ________ years old Margo is currently twice as old as Joy was three decades ago, and Joy is currently three times as old as Margo was two decades ago. How old was Joy when Margo was born?

87. ________ in^2 What is the area of a rhombus with side length 10 inches and diagonal lengths that differ by 4 inches?

88. ________ % Octavio bought a very unpredictable stock. Its value increased by 10% each day on Monday, Tuesday and Wednesday, was unchanged on Thursday, then dropped by 30% on Friday. Octavio mistakenly thought that the 30% loss was offset completely by the three 10% gains. By what percent did the stock in fact decrease for the week? Express your answer to the nearest hundredth.

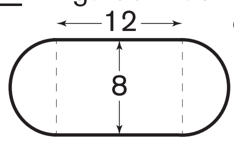
89. ________ inches Tootie Frootie candy is cylindrical in shape and comes in two different sizes. One size is 1.25 inches long with a circumference of 2 inches, and the other is 3.25 inches long and has twice the volume of the first. What is the circumference of the larger size? Express your answer as a decimal to the nearest hundredth.

90. ________ truck-loads A 32-foot by 15-foot by 8-foot hole is being dug in Winnie's yard for a rectangular swimming pool. If the truck hauling away the dirt holds a maximum of 6 cubic yards, how many truckloads of dirt will be taken away? Express your answer as a whole number.

Workout 4

91. ______ quarters Laurie had 30 coins, all dimes and quarters, in a jar. Every morning for five days, she took out a quarter; then she replaced it with a dime when she got home from school. After the five days, Laurie had twice as many dimes as quarters. How many quarters were initially in Laurie's jar?

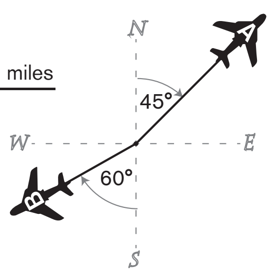
92. ______ yd^3 A garden has the shape of a rectangle with semicircles on either end as shown. The length of the rectangular portion is 12 feet, and the width is 8 feet. How many cubic yards of topsoil are needed to fill the entire garden uniformly to a depth of 2 inches? Express your answer as a decimal to the nearest tenth.



93. ______ Two positive numbers a and b have geometric mean x if $x > 0$ and $\frac{a}{x} = \frac{x}{b}$. If 9 is the geometric mean of m and $m + 3.3$, what is the value of m? Express your answer as a decimal to the nearest tenth.

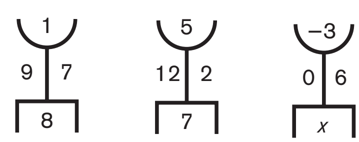
94. ______ mi/h On Oscar's trip to school, he took 50 minutes to walk 2.7 miles. He walked at a constant speed for the first 40 minutes, then increased his speed by exactly 1 mi/h. How fast was Oscar walking for the first 40 minutes of his trip? Express your answer as a decimal to the nearest hundredth.

95. ______ minutes Asanji and Allen are hired to paint the fence around a neighbor's property. Working alone, Allen can paint the entire fence in 9 hours, and Asanji can paint the same fence, working alone, in 7 hours. Working together, how many minutes will it take them to paint two-thirds of the fence? Express your answer as a decimal to the nearest tenth.



96. ______ miles Airplane A flies 45 degrees east of north at a constant speed of 300 mi/h. Traveling at the same altitude as airplane A, airplane B flies 60 degrees west of south at a constant speed of 250 mi/h. Both flights originate at the same time and location. What is the distance between airplanes A and B after they have been flying for 2 hours? Express your answer to the nearest whole number.

97. ______ If the pattern continues, what is the value of x in the third figure?



98. ______ inches The largest circular pizza ever made had a diameter of 122 feet 8 inches. To divide the pizza among 150 contest winners, the chef made straight cuts through the center of the pizza, all equal in length to its diameter. If all the slices had the same area, how many inches were there between slices, measured along the pizza's circumference? Express your answer as a decimal to the nearest tenth.

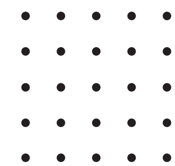
99. ______ If $f(x) = 3x - 4$ and $g(x) = x^2 - 2x + 5$, what is the value of $g(-4) + f(10)$?

100. ______ years The amount of water in a reservoir declines 10% each year. What is the minimum number of whole years it will take until less than half of the original amount of water remains?

Warm-Up 7

101. ________ points In a talent contest involving three finalists, audience members vote for first, second and third place by assigning each a 1, 2 or 3, respectively. When all the votes have been totaled, the finalist with the lowest total wins. If 100 audience members vote and there are no ties in the outcome, what is the greatest number of points that the winner can get?

102. ________ lengths Segments of how many distinct positive lengths can be drawn using pairs of points in this 5 × 5 evenly spaced grid as endpoints?



103. ________ minutes The time it takes a cube of ice to melt at a certain temperature varies directly with the surface area of the cube. If it takes 3 hours for a 1-foot cube of ice to melt, how many minutes will it take a 1-inch cube of ice to melt if the temperature remains constant? Express your answer as a decimal to the nearest hundredth.

104. ________ The mean score on the last test in Ms. McMean's class of 24 students is 88. If the four lowest scores are excluded, the mean is 94. If the four highest scores are excluded, the mean is 86. What is the absolute difference between the means of the four highest and four lowest test scores?

105. ________ feet A ball is released from a height of 16 feet, and each time it strikes the ground, it bounces back to a height one-fourth of its previous height. At the instant when the ball strikes the ground for the fifth time, how many total feet has it traveled since it was released? Express your answer as a mixed number.

106. ________ A passenger jet flies a certain distance in 3 hours 30 minutes when traveling west to east. The same distance from east to west requires 4 hours 15 minutes. What is the ratio of the passenger jet's speed going east to the speed going west? Express your answer as a common fraction.

107. ________ lengths In triangle ABC with acute angle BAC, the lengths of sides AB and BC are 15 units and 16 units, respectively. How many possible integer lengths are there for side AC?

108. ________ If $X \# Y = \dfrac{X}{Y} + XY$, what is the value of 15 # (6 # 2)?

109. ________ % When Jose looked at the clock, he noticed that 20% of the total time from 4:00 p.m. to 5:00 p.m. had elapsed. What percent of the time from 1:00 p.m. to 6:00 p.m. had elapsed, when Jose looked at the clock?

110. ________ combinations In the country of Woodington, three denominations of coins are used to pay for goods and services. Each coin has a value of 1 dollar, 3 dollars or 7 dollars. How many different combinations of coins can be used to pay exactly 15 dollars in Woodington?

Warm-Up 8

111. ________ people At 8:00 a.m., the chairperson of the homecoming committee shared this year's theme with the three other committee members. Within an hour, each of those three committee members told three people who were not on the committee. Every hour after that, each person who had just been told within the previous hour then told three other people who had not yet been told. How many people knew the homecoming theme by the time the first lunch period started at 11:00 a.m.?

112. ________ If $a \blacktriangle b = |a - b|$, then what is the sum of all numbers x such that $(3 \blacktriangle x) \blacktriangle 8 = 2$?

113. ________ integers How many positive integers m are there such that the least common multiple of m and 150 is 300?

114. ________ words Carrick is learning to read. During his first lesson, Carrick read 50 words, and during each lesson thereafter, he read 10 more words than he read during the lesson before. If Carrick had one reading lesson each day for 30 days, what is the total number of words he read in 30 lessons?

115. ________ A standard deck of cards consists of cards numbered 2 through 10 plus a Jack, Queen, King and Ace in each of four different suits. In a particular card game, Jacks, Queens and Kings are each worth 10 points, Aces are worth 11 points and numbered cards are worth face value. Each of four players is dealt three cards, and the winner is the player with the greatest sum of cards of the same suit. If cards have been dealt to the four players as shown, and Austin then is dealt a card from those remaining in the deck, what is the probability that Austin has the winning hand? Express your answer as a common fraction.

AUSTIN	BAILEY	COOPER	DALLAS
8 ♦	Ace ♦	Ace ♥	King ♣
9 ♦	King ♦	3 ♥	9 ♣
?	3 ♠	Queen ♦	5 ♥

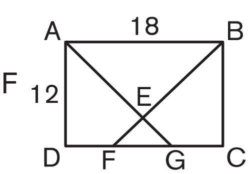
116. ________ If l, m and n are each distinct members of the set $\{2, -\frac{1}{6}, \frac{1}{3}, -3, 12\}$, what is the least possible value of $\frac{l \times m}{n}$?

117. ________ ways In how many ways can each of the digits 3, 5 and 7 be used exactly once to replace X, Y and Z to make the true inequality 0.XY < 0.Z?

118. ________ For a list of eight positive integers, the mean, median, unique mode and range are 8. What is the greatest integer that could be in this set?

119. ________ Two numbers, x and y, each between 0 and 1, are multiplied. If the tenths digit of x is 1 and the tenths digit of y is 2, what is the greatest possible value of the hundredths digit of the product?

120. ________ units² Quadrilateral ABCD, shown here, is a 12×18 rectangle. Segments BF and AG bisect angles ABC and BAD, respectively, and intersect at E. What is the area of $\triangle$EFG?

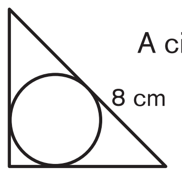


Workout 5

121. ________ inches A photo measures 5 inches wide by 3 inches high. If the total area of the photo is enlarged by 150%, while maintaining the same ratio of width to height, what is the height of the enlarged photo? Express your answer as a decimal to the nearest tenth.

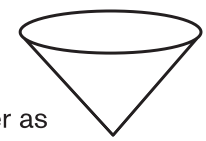
122. ________ The sum of two numbers is 7 and their difference is 18. What is their product? Express your answer as a decimal to the nearest hundredth.

123. ________ cm^2



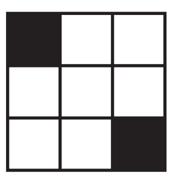
A circle is inscribed in an isosceles right triangle with hypotenuse 8 cm. What is the area of the circle? Express your answer as a decimal to the nearest tenth.

124. ________ A water tank in the shape of a cone, with its point down as shown, has base radius 12 feet and height 8 feet. If the tank is half full by volume, what is the ratio of the depth of the water to the height of the tank? Express your answer as a decimal to the nearest hundredth.



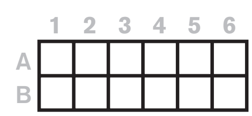
125. ________ inches On her 13th birthday, Moesha's height is exactly 62 inches. If she uses the formula $H(t) = 61 + 2^{0.4t}$ to determine her height H, in inches, t years after her 13th birthday, what should Moesha expect her height to be on her 18th birthday?

126. ________



When filled in correctly with nonzero digits, each row and column in this cross-number grid contains a perfect square. No two rows contain the same perfect square, and no two columns contain the same perfect square. What is the least possible sum of all the digits in the completed grid?

127. ________ ways The figure shows a rectangle divided into 12 congruent squares. In how many ways can each square be colored red, blue or green, so that when the rectangle is rotated 180° every location (for example, row A column 1) contains a square of a different color than the one that was originally in that location?



128. ________ children

Age	Fee
2 & under	No charge
3 to 12	$3 each
62 & older	$7 each
All others	$10 each

On their vacation, the Ship family took a boat ride to Shipwreck Island. The table shows the fees charged by the boat company, based on each passenger's age. The total charge for the 12 family members was $73. If the number who are "others" exceeds the number who are 62 and older, what is the maximum possible number of children ages 3 to 12?

129. ________ What is the greatest possible sum of the reciprocals of two positive integers with a sum of 11? Express your answer as a common fraction.

130. ________ integers How many positive integers less than or equal to 100 have the same number of odd factors as even factors?

Workout 6

131. ________ If a, b and c are integers between -10 and 10 inclusive such that $a^3 + b^3 = c^3$, what is the greatest possible value of a + b + c?

132. ________ The positive integers 1 through 50 are written on 50 cards with one integer on each card. If Matt draws one card at random, what is the probability that the number on the card is a multiple of 6 or 8? Express your answer as a common fraction.

133. ________ units In Figure 1, the vertices of equilateral triangle ABC are connected with segments AB and AC. In Figure 2, the vertices are connected by congruent segments AD, BD and CD that intersect at D. D is the intersection of the medians of triangle ABC. If triangle ABC has side length 1 unit, what is the absolute difference between the sum of the lengths of the two segments in Figure 1 and the sum of the lengths of the three segments in Figure 2? Express your answer in simplest radical form.

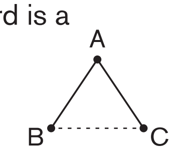


Figure 1

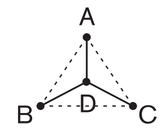


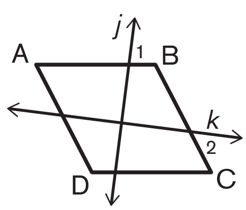
Figure 2

134. ________ % Erik and Alicia went to a restaurant and ordered the same thing for lunch. At the end of the meal, they used different methods to calculate the tip. Erik paid a tip equal to twice the sales tax, and Alicia doubled the final amount on the bill, including the sales tax, and then moved the decimal point in the resulting amount one place to the left. Amazingly, these two methods resulted in the same tip. What was the percent sales tax? Express your answer to the nearest tenth.

135. ________ units^2 The four vertices of a rectangle also are vertices of a regular hexagon of side length 1 unit. What is the area of the rectangle? Express your answer in simplest radical form.

136. ________ % In a certain state, gas prices increased from \$1.72 to \$3.84 per gallon over a 10-year period. If the prices increased uniformly by the same percent from year to year, what was the annual percent increase? Express your answer to the nearest tenth.

137. ________ degrees

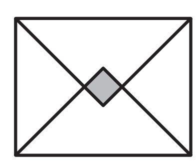


The figure shows parallelogram ABCD with $\angle A$ of measure 57 degrees. Line j is perpendicular to line k and also intersects side AB such that $m\angle 1 = 81$ degrees. What is the $m\angle 2$?

138. \$________ The table shows the charges associated with shipping 1 to 10 items for one particular retailer. Based on this, what is the least amount a customer can pay per item for shipping?

Items	1	2	3	4	5	6	7	8	9	10
Shipping	\$1.85	\$2.15	\$2.65	\$3.35	\$4.25	\$5.35	\$6.65	\$8.15	\$9.85	\$11.75

139. ________ % The side lengths of this rectangle are in the ratio 5:7. Two isosceles right triangles are drawn in the rectangle's interior. The hypotenuse of each of these right triangles is one of the longer sides of the rectangle. The shaded region represents what percent of the area of the rectangle? Express your answer to the nearest tenth.



140. ________ If $a \blacktriangledown b = a \cdot b + 3$, what is the absolute difference between $(10 \blacktriangledown 11) \blacktriangledown 12$ and $10 \blacktriangledown (11 \blacktriangledown 12)$?

Warm-Up 9

141. ________ Let $f(x) = 3x^2 - 7$ and $g(x) = 2x + 5$. What is the absolute difference between $f(g(-2))$ and $g(f(-2))$?

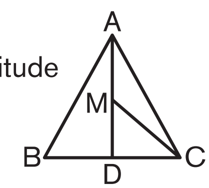
142. ________ What is the value of $1 - 2 + 3 - 4 + 5 - 6 + \cdots + 2013 - 2014 + 2015$?

143. ________ miles Ted and Fred are 60 miles apart and moving toward each other. A carrier pigeon flies back and forth between Ted and Fred without stopping until they meet. If Ted and Fred each maintain a constant speed of 30 mi/h and the pigeon maintains a constant speed of 45 mi/h, what is the total number of miles the pigeon will have flown when Ted and Fred meet?

144. ________ For 12 base x, written 12_x, what is the value of x if $12_x = 3(11_3)$?

145. ________ quadri-laterals How many different quadrilaterals have vertices with integer coordinates (x, y) such that $0 \le x \le 2$ and $0 \le y \le 2$?

146. ________ Equilateral triangle ABC has side length 6 units, and M is the midpoint of altitude AD. The length of segment MC expressed as a common fraction in simplest radical form is $\dfrac{a\sqrt{b}}{c}$ units, where a, b and c are integers. What is the value of $a + b + c$?



147. ________ The perimeter of square A is 28 feet less than the perimeter of square B. The area of square A is 161 square feet less than the area of square B. If x and y are the side lengths of square A and square B, respectively, what is the value of $x + y$?

148. ________ pairings In a certain game, each move consists of pairing tiles of equal value to create a new tile with double the value. For example, a pair of 4-tiles combine to make an 8-tile. Given an unlimited supply of 2-tiles, what is the minimum number of pairings needed to build a tile with a value of 32?

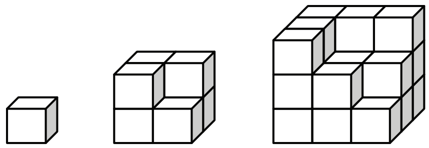
149. ________ The absolute difference between the mean and median of five distinct positive integers is at least 2. If the five integers are 3, 22, 7, 12 and x, with 3 and 22 being the least and greatest values, respectively, what is the sum of all possible values of x?

150. ________ integers A positive integer k is said to be *divisive* if $k > 10$, all digits of k are nonzero and each digit of k, except the units digit, is a proper divisor (any factor of the number except the number itself) of the digit to its immediate right. Based on this, how many positive integers are divisive?

Warm-Up 10

151. ________ players For each of the 20 basketball games this season, Coach Washington needs to choose 5 players to start. If he doesn't want the same 5 players starting together more than once, what is the minimum number of players the coach needs on his team roster?

152. ________ cubes



If this pattern continues, how many cubes will be in the next figure?

153. ________ In a particular arithmetic sequence, the fourth term is 38, and the sum of the second and seventh terms is 85. What is the value of the fifth term of the sequence?

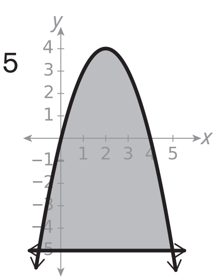
154. ________ units^2 Let O be the point (0, 0) in the coordinate plane, and let A be the point (3, 7). If B is the point obtained by rotating A 90 degrees counterclockwise about O, then what is the area of triangle ABO?

155. ________ pairs How many ordered pairs of integers (x, y) satisfy the equation $x^2 + y^2 = 2500$?

156. ________ Each letter of the alphabet is assigned a numerical value equal to its position, so that A = 1, B = 2, C = 3, …, Z = 26. The value of a word is the sum of the values of its letters. For example, the value of THREE is 20 + 8 + 18 + 5 + 5 = 56. What integer from 1 to 20, inclusive, when spelled out, has the greatest value?

157. ________ ways Spirit Committee members will be chosen from students nominated in each grade. Two students will be chosen from the five nominated 6th graders. Four students will be chosen from the seven nominated 7th graders. Six students will be chosen from the nine nominated 8th graders. In how many different ways can the committee members be chosen?

158. ________ units^2 The area of the region bounded by the parabola $y = 4x - x^2$ and the line $y = -5$ can be determined using the formula $A = \frac{2}{3}bh$, where b is the length of the horizontal base and h is the vertical distance from the vertex of the parabola to the base. What is the area of this region, shown shaded?



159. ________ fizz In the country of Fizzle, coins come in denominations of 5, 8 and 11 fizz. What is the greatest integer amount that cannot be paid with a whole number of these coins?

160. ________ inches An 8-inch by 12-inch paper napkin is folded in half three times, with each fold resulting in a smaller rectangle. What is the longest possible diagonal for the final rectangle? Express your answer in simplest radical form.

Warm-Up 11

161. ________ in^2 A right triangle has sides, in inches, measuring $2n$, $n^2 - 1$ and $n^2 + 1$, where n is a positive integer. What is the area of this triangle if its perimeter is 40 inches?

162. ________ integers How many positive three-digit integers have the property that the tens digit is the sum of the units digit and the hundreds digit?

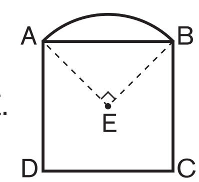
163. ________ An arithmetic sequence of positive integers has a common difference that is three times the first term. If the sum of the first five terms is equal to the absolute difference between the first term and its square, what is the first term in the sequence?

164. ________ in^2 The area of a rectangle is 168 in^2, and its perimeter is 62 inches. What is the product of the lengths of the diagonals of this rectangle?

165. ________ If n_k is the kth digit to the right of the decimal point in the decimal representation of $\dfrac{2240}{1111}$, what is the value of $1000n_{16} + 100n_{13} + 10n_{10} + n_7$?

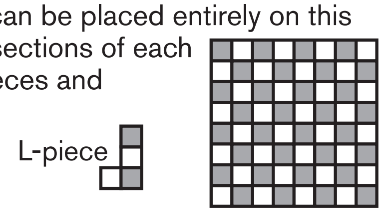
166. ________ What is the greatest prime factor of $6^5 - 4^5 - 2^5$?

167. ________ in^2 Square ABCD, shown here, has side length 4 inches. The arc from A to B is part of a circle whose center coincides with the center of the square, point E. What is the area of the entire figure? Express your answer in terms of π.



168. ________ quadrants The graph of $f(x) = -x^2 + 8x - 15$ contains points in how many quadrants of the Cartesian coordinate plane?

169. ________ L-pieces What is the maximum number of the L-pieces shown that can be placed entirely on this 8 × 8 board, with none overlapping, such that the shaded sections of each L-piece are on shaded squares of the board? (Rotating pieces and reflecting pieces are permitted.)



170. ________ steps Justin and Shelby board an escalator as it is moving down. Justin walks down 30 steps and reaches the bottom in 72 seconds, while Shelby walks down 40 steps and reaches the bottom in 60 seconds. If the escalator weren't moving, how many steps would Justin and Shelby each have to walk down to reach the bottom?

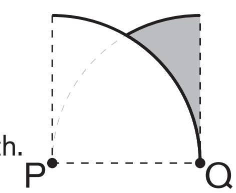
Workout 7

171. ________ If $y = -\dfrac{5}{4}x + 3$ and $y = -\dfrac{5}{4}x + 11$ are equidistant from a point with coordinates $(t, 14)$, what is the value of t? Express your answer as a decimal to the nearest tenth.

172. $________ Priya needs to purchase a camera and a printer. Priya has two coupons, one for 15% off a single item and the other for 20% off any other single item. Priya determines that applying the 20% discount to the printer and the 15% discount to the camera would save her $0.99 more than if she applied the 15% coupon to the printer and the 20% discount to the camera. What is the absolute difference between the price of the camera and the price of the printer?

173. ________% The Lemurs are playing the Capybaras in a series of three baseball games. The games alternate between the Lemurs' home field and the Capybaras' home field, with the first game taking place at the Lemurs' field. Each team has a 60% chance of winning a game on its home field. What is the probability, expressed as a percent, that the Lemurs will win at least two of the three games? Express your answer to the nearest tenth.

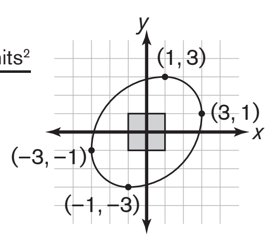
174. ________units^2 The figure shows a quarter of unit circle P and a quarter of unit circle Q drawn with the center of each on the circumference of the other. What is the area of the shaded region? Express your answer as a decimal to the nearest hundredth.



175. ________ For a particular sequence, if $a_1 = 2$ and $a_{n+1} = -a_n + 2n$ for $n \geq 1$, what is a_{2016}?

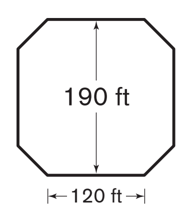
176. ________ A math class is 60% girls. After g girls and 1 boy join the class, it is 75% girls. What is the least possible value of g?

177. ________units^2



The figure shows a square with vertices (1, 1), (−1, 1), (−1, −1) and (1, −1) and a curve made from four quarter-circular arcs, each centered at a vertex of the square. If the endpoints of the four arcs are (3, 1), (1, 3), (−3, −1) and (−1, −3), what is the area inside the curve? Express your answer in terms of π.

178. ________feet This octagon is classified as *isogonal* since each of its vertices is between one short side and one long side, and its interior angles are all congruent. If the long sides have length 120 feet and the distance between two parallel long sides is 190 feet, what is the perimeter of the octagon? Express your answer to the nearest whole number.



179. ________terms All terms of an arithmetic sequence are integers. If the first term is 13, the last term is 77 and the sequence has n terms, what is the median of all possible values of n?

180. ________ Will and Ian each randomly choose a positive divisor of 20. What is the probability that the least common multiple of their chosen numbers is 20? Express your answer as a common fraction.

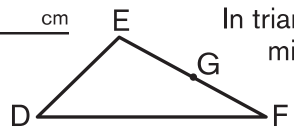
Workout 8

181. ________ divisors How many positive integer divisors does 2016 have?

182. ________ If $f(x) = x^x$ and $f(f(f(2))) = 2^k$, what is the integer value of k?

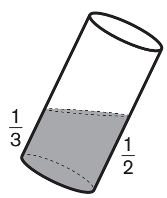
183. ________ units² Circle O, with radius 8 units, and circle W, with radius 3 units, are externally tangent. The circles are also tangent to line m at distinct points B and L, respectively. What is the area of quadrilateral BOWL? Express your answer in simplest radical form.

184. ________ cm In triangle DEF, DE = 6 cm, DF = 9 cm and DG = 7 cm, where G is the midpoint of side EF. What is the length of side EF? Express your answer in simplest radical form.



185. ________ The digits 1 through 7 are each used once to write three prime numbers. Two of these primes have two digits each and one has three digits. What is the greatest possible value of the three-digit prime number?

186. ________ in³ Dan tilts his soda can until the soda is halfway up the can at its highest point and a third of the way up the can at its lowest point. If the can is 4.5 inches tall with a 2.5-inch diameter, how many cubic inches of soda are in the can? Express your answer as a decimal to the nearest tenth.

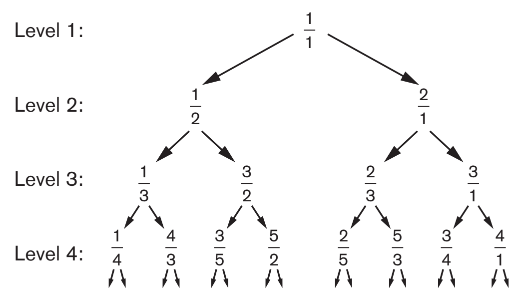


187. ________ triangles Four rows from a sheet of isometric dot paper, where the rows contain three, two, three and two points, respectively, are shown. How many distinct non-equilateral triangles can be drawn with each vertex at a point in this array?

188. ________ Two positive integers not exceeding 100 have the property that their sum is divisible by their difference. What is the greatest possible value of their difference?

189. ________ If Pascal's Triangle is folded along its vertical line of symmetry and then unfolded, what is the sum of the seven numbers on the fold line that are of least value?

190. ________ The figure shows part of an infinite tree diagram in which each fraction $\frac{a}{b}$ has the fractions $\frac{a}{a+b}$ and $\frac{a+b}{b}$ written below it, starting at Level 1 with $\frac{1}{1}$. For what integer n does Level n contain the fraction $\frac{31}{4159}$?



Warm-Up 12

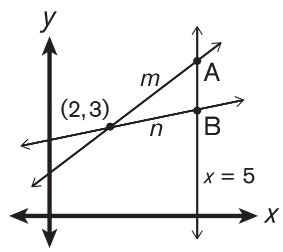
191. ________ When three fair dice are rolled, what is the probability that the product of the three numbers rolled is a prime number? Express your answer as a common fraction.

192. ________ inches A tree is 8 feet tall when planted. After one year, its height has increased by one-sixth of its original height. At the end of the second year, its height is 37.5% greater than its original height. How many inches did the tree grow during the second year?

193. ________ Kenyatta has some quarters on a table, each showing either heads or tails. Kenyatta chooses 20% of the quarters that show heads and 10% of the quarters that show tails and then turns over each of the chosen quarters one time. After she does this, exactly half of the quarters on the table show heads. Before Kenyatta turned the quarters over, what was the ratio of the number of quarters that showed heads to the number that showed tails? Express your answer as a common fraction.

194. ________ minutes If three machines can fill 80 boxes in 2 hours, how many minutes will it take five machines to fill 150 boxes?

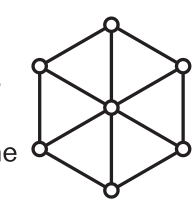
195. ________ units Lines m and n both pass through the point (2, 3) as shown. Line m has slope $\frac{3}{4}$, and line n has slope $\frac{1}{5}$. If lines m and n intersect the line $x = 5$ at the points A and B, respectively, what is the distance AB? Express your answer as a common fraction.



196. ________ What integer is closest in value to $\dfrac{1}{\sqrt{102} - \sqrt{98}}$?

197. $ ________ Victor pays $1.00 for a bottle of soda at his local market. Each time Victor returns three empty bottles, he earns a free bottle of soda. What is the minimum amount Victor must spend to get 10 bottles of soda?

198. ________ ways The figure shows a regular hexagon and seven circles, one at each vertex and another at the hexagon's center. Three diagonals are drawn through the center of the hexagon, connecting opposite vertices. In how many ways can each of the circles be colored either red or blue so that no three collinear circles are the same color?



199. ________ Four husband-and-wife couples attend a show. From these four couples, two people are randomly selected when the performer asks for volunteers from the audience. What is the probability that the two who are selected are a married couple? Express your answer as a common fraction.

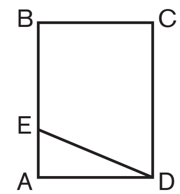
200. ________ In a quadratic equation $Ax^2 + Bx + C = 0$, A, B and C are integers whose only common factor is 1. The roots of the equation are $\frac{2}{3}$ and 4. If $A > 0$, what is the value of $A + B + C$?

Warm-Up 13

201. ________ minutes For the first 4 miles of a 6-mile race, it took Jenny an average of $10\frac{1}{3}$ minutes to run each mile. If she wants to run the entire race in exactly 1 hour, how many minutes, on average, must she take to run each mile for the remainder of the race? Express your answer as a mixed number.

202. ________ hours Two 12-hour clocks were started simultaneously, both showing the same time. Clock A loses 15 minutes each hour, and clock B gains 15 minutes each hour. How many hours pass before the first instance when Clock A again shows the same time as Clock B?

203. ________ What is the value of $f(f(f(19) + 1) + 1)$ if $f(x) = \begin{cases} x^2 & \text{if } x \text{ is even} \\ \dfrac{x-3}{2} & \text{if } x \text{ is odd} \end{cases}$?



204. ________ Rectangle ABCD, shown here, has point E on side AB such that the ratio of the area of trapezoid EBCD to that of triangle AED is 5:1. What is the value of the ratio AE:EB? Express your answer as a common fraction.

205. ________ in^3 Kendra will make a box with an open top by cutting, folding and taping a 12-inch by 16-inch rectangular piece of cardboard. She will begin by cutting from each corner of the flat cardboard a square with a side length that is a multiple of 0.5 inch. What is the maximum possible volume of the box? Express your answer as a decimal to the nearest tenth.

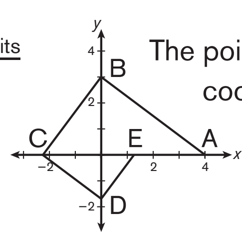
206. ________ What is the sum of the coefficients of the terms when $(2x + y)^5$ is expanded?

207. ________ In a certain game of darts, a dart that lands on the bull's-eye scores 11 points, and a dart that lands any other place on the board scores 7 points. A player throws three darts, and the probability that the player hits the bull's-eye is 50% on each throw, independent of previous results. Given that each dart lands on the board, and at least one dart lands on the bull's-eye, what is the probability that the total number of points scored will be prime? Express your answer as a common fraction.

208. ________ integers How many 4-digit integers have their digits in strictly ascending order?

209. (____ , ____) P(−3, −2) is reflected over the line $y = -x$ and then translated 4 units right and 1 unit down. What are the coordinates of the final image of P? Express your answer as an ordered pair.

210. ________ units



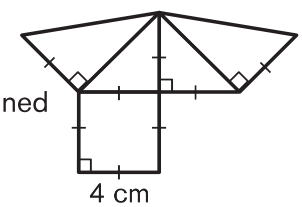
The points A(4, 0), B(0, 3), C(c, 0), D(0, d) and E(e, 0) are graphed on the coordinate plane as shown. If $\overline{AB} \perp \overline{BC}$, $\overline{BC} \perp \overline{CD}$ and $\overline{CD} \perp \overline{DE}$, what is the combined length of the four segments? Express your answer as a common fraction.

Warm-Up 14

211. ________ If $3^x = y^2$, and $9^{5x} = y^z$, what is the value of z?

212. ________ What is the value of the sum $\frac{3+6}{9} + \frac{12+15}{18} + \frac{21+24}{27} + \cdots + \frac{48+51}{54}$? Express your answer as a common fraction.

213. ________ members Namakshi wants to arrange her marching band in a rectangular array with the same number of band members in each row. But when she tried putting them in 3 rows, she had 1 band member left over; in 5 rows, she had 2 members left over; in 7 rows, she had 3 members left over; and in 9 rows, she had 4 members left over. What is the least number of members that could be in the band?

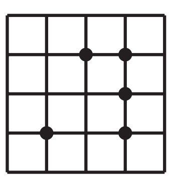


214. ________ cm^3 Three pyramids, each created using the net shown here, can be combined to form a cube. What is the volume of the cube?

215. ________ In a certain sequence of numbers, every term after the second term is equal to the sum of the two preceding terms. If the 99th term of the sequence is 7 and the 102nd term is 69, what is the 100th term of the sequence?

216. ________ cents Stacey has some pennies, nickels and dimes divided between her left and right pockets. She has the same number of coins in each pocket, and the coins in her two pockets add to the same total value. Stacey does not have the same number of pennies in each pocket. What is the minimum total value that she could have in both pockets combined?

217. ________ units



In this grid composed of 16 unit squares, the five dots can be labeled with the letters A through E so that AB < BC < CD < DE. What is the length of segment CE?

218. ________ What is the probability that a randomly chosen two-digit number containing at least one 1 or 9 is prime? Express your answer as a common fraction.

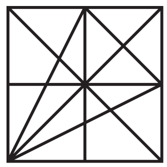
219. ________ minutes Alfonso, Benjamin and Carmen walk at speeds of 2, 6 and 8 feet per second, respectively. The three friends start walking together at the same time in the same direction around an oval track that measures 440 yards around. After how many minutes are the three friends next at the same exact location on the track at the same exact time?

220. ________ For each positive integer $n \leq 55$, if $S(n)$ is the sum of the positive integer divisors of n, what is the greatest possible value of $S(n)$?

Counting Stretch

221.________ Hazel wrote the integers 1 through 321 on the board. How many total digits did she write?

222.________ triangles



How many triangles of any size are in this figure?

223.________ ways In how many ways can one knife, one fork and one spoon be distributed, in any order, to three people, if each person is given 0, 1, 2 or 3 utensils?

224.________ ways Using pennies, nickels, dimes and quarters, how many ways can you make 67 cents?

225.________ scores In the game Fortrix, a player can earn 3, 7 or 11 points on a turn. How many different scores are possible for a single player after six turns?

226.________ integers How many 3-digit integers are divisible by both 5 and 17?

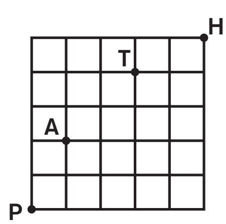
227.________ integers How many positive integers less than 40 are relatively prime to both 7 and 10?

228.________ palin-dromes How many palindromes are between 9 and 1009?

229.________ paths In the 3×3 grid shown, a path can begin in any cell and can pass through a cell more than once. How many such paths spell ROTOR?

R	O	R
O	T	O
R	O	R

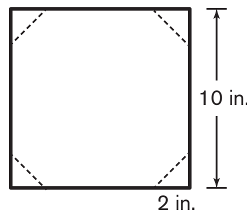
230.________ paths



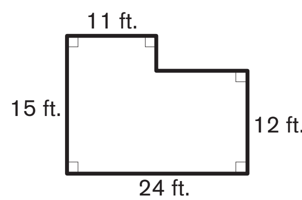
Moving only up and right, how many paths from P to H pass through A and T?

Area Stretch

231.________% Norm has a square sheet of paper with 10-inch sides. Along each side, he makes a mark 2 inches from each corner. He then draws a line segment connecting the two marks near each corner. Finally, he cuts along each line segment, removing a triangle from each corner of the square and creating an octagon. What percentage of the area of the square is the area of the octagon?



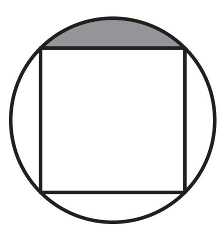
232.________ft² The figure shows an office floor plan. How many square feet does this office occupy?



233.________m² A running track consists of two parallel straight segments, each 100 meters long, connected by two semicircular stretches, each with inner diameter 50 meters. What is the total area enclosed by the running track? Express your answer to the nearest hundred.

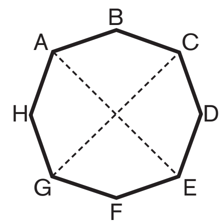
234.________units² What is the greatest possible area of a concave pentagon in the coordinate plane with vertices (−2, 0), (2, 0), (2, 10), (0, 6) and (−2, 10)?

235.________units² A square is inscribed in a circle of radius 4 units. The square divides the interior of the circle into five regions, four of which lie outside the square. What is the area of the shaded region? Express your answer in terms of π.

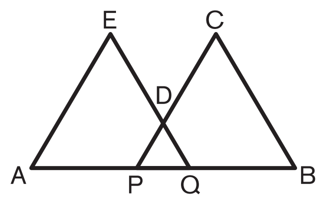


236.________ in^2 Amy marks two points A and B that are 4 inches apart. She draws one circle that has segment AB as a diameter. She then draws a larger circle, which overlaps the first circle, such that the arc from A to B along its circumference is a quarter-circle. What is the total area covered by the two circles? Express your answer in terms of π.

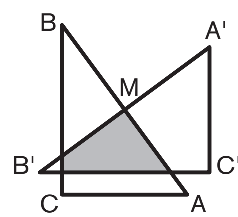
237.________ units^2 In convex octagon ABCDEFGH, shown here, each side has length 6 units, and diagonals AE and CG have length 16 units. If the octagon is symmetric across both diagonals AE and CG, what is its area? Express your answer in simplest radical form.



238.________ units^2 In this figure, AE = EQ = BC = CP = 10 units, and AQ = BP = 12 units. The points A, P, Q and B are collinear. If the perimeter of the concave pentagon ABCDE is 52 units, what is its area? Express your answer as a common fraction.



239.________ units^2 Right triangle ABC with AC = 3 units, BC = 4 units and AB = 5 units is rotated 90° counterclockwise about M, the midpoint of side AB, to create a new right triangle A'B'C'. What is the area of the shaded region where triangles ABC and A'B'C' overlap? Express your answer as a common fraction.



240.________ units In right triangle ABC, ∠C is a right angle, AC = 10 units and BC = 24 units. If a point X is located inside triangle ABC so that the distance from X to side AB is twice the distance from X to side AC, and the distance from X to side AC is twice the distance from X to side BC, what is the distance from X to side AB? Express your answer as a common fraction.

Modular Arithmetic Stretch

Modular arithmetic is a system of integer arithmetic that enables us obtain information and draw conclusions about large quantities and calculations. It would be extremely helpful, for instance, when asked to find the units digit of 2^{2015} if we didn't really have to calculate the value of the expression to get that information. Modular arithmetic allows us to do just that!

THE BASICS:

The simplest example of modular arithmetic is commonly referred to as "clock arithmetic." Suppose it is 3 o'clock now and I want to know what time it will be in 145 hours. We could count from 3 o'clock for 145 consecutive hours. We certainly wouldn't be expected to count 145 hours starting with 3 o'clock. Suppose we did counting the hours from 3 o'clock. What happens when we get to 12 o'clock? We continue counting but begin a new 12-hour cycle. Instead of counting 145 hours, we can just see how many of these 12-hour cycles we'd go through counting 145 hours. More importantly, we need to determine how many hours would remain after making it through the last full 12-hour cycle.

In this example, the value 12 is called the ***modulus*** and what is left over is called the remainder. In this case, we can determine fairly quickly that there are 12 full 12-hour cycles in 145 hours, with a remainder of 1 hour (since $12 \times 12 = 144$ and $145 - 144 = 1$).

Standard arithmetic: $145 = 12 \times 12 + 1$

Modular arithmetic we write: $145 \equiv 1 \pmod{12}$ *Read **"145 is congruent to 1 modulo 12"***

The remainder of 1 tells me that it will be the same time 145 hours after 3 o'clock that it will be 1 hour after 3 o'clock. And that time is 4 o'clock.

Here's another example of modular arithmetic. Suppose today is Tuesday. What day of the week will it be 417 days from now? Since the days of the week are on a 7-day repeating cycle, the modulus here is 7. If we divide 417 by 7, we get

Standard arithmetic: $417 = 59 \times 7 + 4$

Modular arithmetic we write: $417 \equiv 4 \pmod{7}$

Thus, 417 days from Tuesday will be the same day of the week as 4 days from Tuesday, Saturday.

TRY THESE

241.__________ If the current month is July, what month will it be in 152 months?

242.__________ a.m. / p.m. If the time is currently 8 a.m., what time will it be in 255 hours? *Circle a.m. or p.m. in answer blank.*

243.__________ m Jennie goes out every morning and jogs on the school track. The track is 400 meters around. If Jennie runs 5310 meters then how far will she be from where she started once she finished her run?

Modular Addition: What is the remainder when 9813 + 7762 + 11252 is divided by 10?

$$9813 + 7762 + 11252 = (981 \times 10 + 3) + (776 \times 10 + 2) + (1125 \times 10 + 2)$$
$$= (981 + 776 + 1125) \times 10 + (3 + 2 + 2)$$

Since we are only interested in the remainder, we need only focus on the last part. We see that the remainder is 3 + 2 + 2 = **7**. Written in modular arithmetic notation it would look like this:

$$9813 + 7761 + 11252 \equiv 3 + 2 + 2 \equiv 7 \pmod{10}$$

Modular Multiplication: What is the remainder whsen 9813×7762 is divided by 10?

$$9813 \times 7762 = (981 \times 10 + 3) \times (776 \times 10 + 2)$$
$$= (981 \times 776 \times 10^2) + (981 \times 2 \times 10) + (776 \times 3 \times 10) + (3 \times 2)$$

The first three terms are multiples of 10, and once again last term is the remainder $3 \times 2 =$ **6**. Written in modular arithmetic notation would look like this:

$$9813 \times 7762 \equiv 3 \times 2 \equiv 6 \pmod{10}$$

More Mod Shortcuts: There are many useful applications of modular arithmetic. Here are just a few more.

- Consider the powers of 3: $3^0 = 1$; $3^1 = 3$; $3^2 = 9$; $3^3 = 27$; $3^4 = 81$; $3^5 = 243$; $3^6 = 729$
  Notice that the units digits are repeated every four powers of 3, so the modulus is 4. Repeating units digits correspond to remainders 1, 2, 3 and 0.
- Suppose you want the unit digit of 3^{53}. First, we note that $53 \equiv 1 \pmod 4$ since the remainder 1 corresponds to units digit 3, thus, the expansion of 3^{53} has a units digit of 3.
- The smallest number that has remainder 1 when divided by 2 and 3 is 7. Why?
  $1 \equiv 7 \pmod 2$ and $1 \equiv 7 \pmod 3$

MODULAR ARITHMETIC PRACTICE

244.__________ What is the last digit of 2^{2015}?

245.__________ What is the value of 122×71 modulo 11?

246.__________ What is the remainder when $5981 \times 8162 \times 476$ is divided by 5?

247.__________ Jon has 29 boxes of donuts with 51 donuts in each box. He wants to divide them into groups of a dozen each. Once he groups them again, how many donuts will be left over?

248.__________ What is the least integer greater than 6 that leaves a remainder of 6 when it is divided by 7 and by 11?

249.__________ When organizing her pencils, Faith notices that when she puts them in groups of 3, 4, 5, or 6, she always has exactly one pencil left over. If Faith has between 10 and 100 pencils, how many pencils does she have?

250.__________ When organizing her pens, Faith notices that when she puts them in groups of 3, 4, 5, or 6, she is always one pen short of being able to make full groups. If Faith has between 10 and 100 pens, how many pens does she have?

MATHCOUNTS COMPETITION SERIES EST. 1983

OFFICIAL RULES + PROCEDURES

The following rules and procedures govern all MATHCOUNTS competitions. The MATHCOUNTS Foundation reserves the right to alter these rules and procedures at any time. **Coaches are responsible for being familiar with the rules and procedures outlined in this handbook.** Coaches should bring any difficulty in procedures or in student conduct to the immediate attention of the appropriate chapter, state or national official. Students violating any rules may be subject to immediate disqualification.

Any questions regarding the MATHCOUNTS Competition Series Official Rules + Procedures articulated in this handbook should be addressed to the MATHCOUNTS national office at (703) 299-9006 or info@mathcounts.org.

REGISTRATION

The fastest and easiest way to register for the MATHCOUNTS Competition Series is online at www.mathcounts.org/compreg.

For your school to participate in the MATHCOUNTS Competition Series, a school representative is required to complete a registration form and pay the registration fees. A school representative can be a teacher, administrator or parent volunteer who has received expressed permission from his/her child's school administration to register. By completing the Competition Series Registration Form, the coach attests to the school administration's permission to register students for MATHCOUNTS.

School representatives can register online at www.mathcounts.org/compreg or download the Competition Series Registration Form and mail, fax or email a scanned copy of it to the MATHCOUNTS national office. Refer to the Critical 2015-2016 Dates on pg. 11 of this handbook for contact information.

WHAT REGISTRATION COVERS: Registration in the Competition Series entitles a school to:

1) send 1-10 student(s)—depending on number registered—to the Chapter Competition. *Students can advance beyond the chapter level, but this is determined by their performance at the competition.*
2) receive the School Competition Kit, which includes the *2015-2016 MATHCOUNTS School Handbook*, one recognition ribbon for each registered student, 10 student participation certificates and a catalog of additional coaching materials. *Mailings of School Competition Kits will occur on a rolling basis through December 31, 2015.*
3) receive online access to the 2016 School Competition, along with electronic versions of other competition materials at www.mathcounts.org/coaches. *Coaches will receive an email notification no later than November 2, 2015 when the 2016 School Competition is available online.*

Your state or chapter coordinator will be notified of your registration, and then you will be informed of the date and location of your Chapter Competition. **If you have not been contacted by mid-January with competition details, it is your responsibility to contact your local coordinator** to confirm that your registration has been properly routed and that your school's participation is expected. Coordinator contact information is available at www.mathcounts.org/findmycoordinator.

DEADLINES: The sooner your Registration Form is received, the sooner you will receive your preparation materials. To guarantee your school's participation, submit your registration by one of the following deadlines:

Early Bird Discount Deadline: November 13, 2015	Online registrations: submitted by 11:59 PST Emailed or faxed forms: received by 11:59 PST Mailed forms: postmarked by November 13, 2015
Regular Registration Deadline: December 11, 2015*	Online registrations: submitted by 11:59 PST Emailed or faxed forms: received by 11:59 PST Mailed forms: postmarked by December 11, 2015

*Late Registrations may be accepted at the discretion of the MATHCOUNTS national office and your local coordinators, but are not guaranteed. If a school's late registration is accepted, an additional $20 processing fee will be assessed.

REGISTRATION FEES: The cost of your school's registration depends on when your registration is postmarked/emailed/faxed/submitted online. The cost of your school's registration covers the students for the entire Competition Series; there are no additional registration fees to compete at the state or national level. Title I schools (as affirmed by a school's administration) receive a 50% discount off the total cost of their registration.

Number of Registered Students	Registration Postmarked by 11/13/2015	Postmarked between 11/13/2015 and 12/11/2015	Postmarked after 12/11/2015 (+ Late Fee)
1 individual	$25	$30	$50
2 ind.	$50	$60	$80
3 ind.	$75	$90	$110
1 team of 4	$90	$100	$120
1 tm. + 1 ind.	$115	$130	$150
1 tm. + 2 ind.	$140	$160	$180
1 tm. + 3 ind.	$165	$190	$210
1 tm. + 4 ind.	$190	$220	$240
1 tm. + 5 ind.	$215	$250	$270
1 tm. + 6 ind.	$240	$280	$300

ELIGIBILITY REQUIREMENTS

Eligibility requirements for the MATHCOUNTS Competition Series are different from other MATHCOUNTS programs. Eligibility for the National Math Club or the Math Video Challenge does not guarantee eligibility for the Competition Series.

WHO IS ELIGIBLE:

- U.S. students enrolled in the 6th, 7th or 8th grade can participate in MATHCOUNTS competitions.
- Schools that are the students' official school of record can register.
- Any type of school, of any size, can register—public, private, religious, charter, virtual or homeschools—but virtual and homeschools must fill out additional forms to participate (see pgs. 47-48).
- Schools in 50 U.S. states, District of Columbia, Guam, Puerto Rico and Virgin Islands can register.
- Overseas schools that are affiliated with the U.S. Departments of Defense and State can register.

WHO IS NOT ELIGIBLE:

- Students who are not full-time 6th, 7th or 8th graders cannot participate, even if they are taking middle school math classes.
- Academic centers, tutoring centers or enrichment programs that do not function as students' official school of record cannot register. *If it is unclear whether your educational institution is considered a school, please contact your local Department of Education for specific criteria governing your state.*
- Schools located outside of the U.S. states and territories listed on the previous page cannot register.
- Overseas schools not affiliated with the U.S. Departments of Defense or State cannot register.

NUMBER OF STUDENTS ALLOWED: A school can register a maximum of one team of four students and six individuals; these 1-10 student(s) will represent the school at the Chapter Competition. Any number of students can participate at the school level. Prior to the Chapter Competition, coaches must notify their chapter coordinator of which students will be team members and which students will compete as individuals.

NUMBER OF YEARS ALLOWED: Participation in MATHCOUNTS competitions is limited to 3 years for each student, but there is no limit to the number of years a student may participate in school-based coaching.

WHAT TEAM REGISTRATION MEANS: Members of a school team will participate in the Target, Sprint and Team Rounds. Members of a school team also will be eligible to qualify for the Countdown Round (where conducted). Team members will be eligible for team awards, individual awards and progression to the state and national levels based on their individual and/or team performance. It is recommended that your strongest four Mathletes form your school team. Teams of fewer than four will be allowed to compete; however, the team score will be computed by dividing the sum of the team members' scores by four (see pg. 51), meaning, teams of fewer than four students will be at a disadvantage. *Only one team (of up to four students) per school is eligible to compete.*

WHAT INDIVIDUAL REGISTRATION MEANS: Students registered as individuals will participate in the Target and Sprint Rounds, but not the Team Round. Individuals will be eligible to qualify for the Countdown Round (where conducted). Individuals also will be eligible for individual awards and progression to the state and national levels. A student registered as an "individual" may not help his/her school's team advance to the next level of competition. *Up to six students may be registered in addition to or in lieu of a school team.*

HOW STUDENTS ENROLLED PART-TIME AT TWO SCHOOLS PARTICIPATE: *A student may compete only for his/her official school of record.* A student's school of record is the student's base or main school. A student taking limited course work at a second school or educational center may not register or compete for that second school or center, even if the student is not competing for his/her school of record. MATHCOUNTS registration is not determined by where a student takes his or her math course. If there is any doubt about a student's school of record, the chapter or state coordinator must be contacted for a decision before registering.

HOW SMALL SCHOOLS PARTICIPATE: MATHCOUNTS does not distinguish between the sizes of schools for Competition Series registration and competition purposes. Every "brick-and-mortar" school will have the same registration allowance of up to one team of four students and/or up to six individuals. A school's participants may not combine with any other school's participants to form a team when registering or competing.

HOW HOMESCHOOLS PARTICIPATE: Homeschools and/or homeschool groups in compliance with the homeschool laws of the state in which they are located are eligible to participate in MATHCOUNTS competitions in accordance with all other rules. Homeschool coaches must complete the 2015-2016 Homeschool + Virtual School Participation Form, verifying that students from the homeschool or homeschool group are in the 6th, 7th or 8th grade and that each homeschool complies with applicable state laws. Forms can be downloaded at www.mathcounts.org/competition and must be submitted to the MATHCOUNTS national office in order for registrations to be processed.

HOW VIRTUAL SCHOOLS PARTICIPATE: Virtual schools that want to register must contact the MATHCOUNTS national office by December 1, 2015 for specific registration details. Any student registering as a virtual school student must compete in the MATHCOUNTS Chapter Competition assigned according to the student's home address. Additionally, virtual school coaches must complete the 2015-2016 Homeschool + Virtual School Participation Form, verifying that the students from the virtual school are in the 6th, 7th or 8th grade and that the virtual school complies with applicable state laws. Forms must be submitted to the national office in order for registrations to be processed; forms can be downloaded at www.mathcounts.org/competition.

WHAT IS DONE FOR SUBSTITUTIONS OF STUDENTS: Coaches determine which students will represent the school at the Chapter Competition. Coaches cannot substitute team members for the State Competition unless a student voluntarily releases his/her position on the school team. Additional requirements and documentation for substitutions (such as requiring parental release or requiring the substitution request be submitted in writing) are at the discretion of the State Coordinator. A student being added to a team need not be a student who was registered for the Chapter Competition as an individual. Coaches cannot make substitutions for students progressing to the State Competition as individuals. At all levels of competition, student substitutions are not permitted after on-site competition registration has been completed.

WHAT IS DONE FOR RELIGIOUS OBSERVANCES: A student who is unable to attend a competition due to religious observances may take the written portion of the competition up to one week in advance of the scheduled competition. In addition, all competitors from that student's school must take the Sprint and Target Rounds at the same earlier time. If the student who is unable to attend the competition due to a religious observance: (1) is a member of the school team, then the team must take the Team Round at the same earlier time; (2) is not part of the school team, then the team has the option of taking the Team Round during this advance testing or on the regularly scheduled day of the competition with the other school teams. The coordinator must be made aware of the team's decision before the advance testing takes place. *Advance testing will be done at the discretion of the chapter and state coordinators. If advance testing is deemed possible, it will be conducted under proctored conditions.* Students who qualify for an official Countdown Round but are unable to attend will automatically forfeit one place standing.

WHAT IS DONE FOR STUDENTS WITH SPECIAL NEEDS: Reasonable accommodations may be made to allow students with special needs to participate. However, many accommodations that are employed in a classroom or teaching environment cannot be implemented in the competition setting. Accommodations that are not permissible include, but are not limited to: granting a student extra time during any of the competition rounds or allowing a student to use a calculator for the Sprint or Countdown Rounds. *A request for accommodation of special needs must be directed to chapter or state coordinators in writing at least three weeks in advance of the Chapter or State Competition.* This written request should thoroughly explain a student's special need, as well as what the desired accommodation would entail. In conjunction with the MATHCOUNTS Foundation, coordinators will review the needs of the student and determine if any accommodations will be made. In making final determinations, the feasibility of accommodating these needs at the National Competition will be taken into consideration.

LEVELS OF COMPETITION

There are four levels in the MATHCOUNTS Competition Series: school, chapter (local), state and national. Competition questions are written for 6th, 7th and 8th graders. The competitions can be quite challenging, particularly for students who have not been coached using MATHCOUNTS materials. All competition materials are prepared by the national office.

SCHOOL COMPETITIONS (TYPICALLY HELD IN JANUARY 2016): After several months of coaching, schools registered for the Competition Series should administer the 2016 School Competition to all interested students. The School Competition should be an aid to the coach in determining competitors for the Chapter Competition. _Selection of team and individual competitors is entirely at the discretion of the coach and does not need to be based solely on School Competition scores._ School Competition materials are sent to the coach of a school, and it may be used by the teachers and students only in association with that school's programs and activities. The current year's School Competition questions must remain confidential and may not be used in outside activities, such as tutoring sessions or enrichment programs with students from other schools. For updates or edits, please check www.mathcounts.org/coaches before administering the School Competition.

It is important that the coach look upon coaching sessions during the academic year as opportunities to develop better math skills in all students, not just in those students who will be competing. Therefore, it is suggested that the coach postpone selection of competitors until just prior to the Chapter Competition.

CHAPTER COMPETITIONS (HELD FROM JAN. 30 – FEB. 29, 2016): The Chapter Competition consists of the Sprint, Target and Team Rounds. The Countdown Round (official or just for fun) may or may not be conducted. The chapter and state coordinators determine the date and location of the Chapter Competition in accordance with established national procedures and rules. Winning teams and students will receive recognition. The winning team will advance to the State Competition. Additionally, the two highest-ranking competitors not on the winning team (who may be registered as individuals or as members of a team) will advance to the State Competition. This is a minimum of six advancing Mathletes (assuming the winning team has four members). Additional teams and/or individuals also may progress at the discretion of the state coordinator, but the policy for progression must be consistent for all chapters within a state.

STATE COMPETITIONS (HELD FROM MAR. 1-31, 2016): The State Competition consists of the Sprint, Target and Team Rounds. The Countdown Round (official or just for fun) may or may not be included. The state coordinator determines the date and location of the State Competition in accordance with established national procedures and rules. Winning teams and students will receive recognition. The four highest-ranked Mathletes and the coach of the winning team from each State Competition will receive an all-expenses-paid trip to the National Competition.

2016 RAYTHEON MATHCOUNTS NATIONAL COMPETITION (HELD MAY 8-9 IN WASHINGTON, DC): The National Competition consists of the Sprint, Target, Team and Countdown Rounds (conducted officially). Expenses of the state team and coach to travel to the National Competition will be paid by MATHCOUNTS. The national program does not make provisions for the attendance of additional students or coaches. All national competitors will receive a plaque and other items in recognition of their achievements. Winning teams and individuals also will receive medals, trophies and college scholarships.

COMPETITION COMPONENTS

The four rounds of a MATHCOUNTS competition, each described below, are designed to be completed in approximately three hours:

TARGET ROUND (approximately 30 minutes): In this round eight problems are presented to competitors in four pairs (six minutes per pair). The multi-step problems featured in this round engage Mathletes in mathematical reasoning and problem-solving processes. _Problems assume the use of calculators._

SPRINT ROUND (40 minutes): Consisting of 30 problems, this round tests accuracy, with the time period allowing only the most capable students to complete all of the problems. _Calculators are not permitted._

TEAM ROUND (20 minutes): In this round, interaction among team members is permitted and encouraged as they work together to solve 10 problems. _Problems assume the use of calculators._

Note: The order in which the written rounds (Target, Sprint and Team) are administered is at the discretion of the competition coordinator.

COUNTDOWN ROUND: A fast-paced oral competition for top-scoring individuals (based on scores on the Target and Sprint Rounds), this round allows pairs of Mathletes to compete against each other and the clock to solve problems. *Calculators are not permitted.*

At Chapter and State Competitions, a Countdown Round (1) may be conducted officially, (2) may be conducted unofficially (for fun) or (3) may be omitted. However, the use of an official Countdown Round must be consistent for all chapters within a state. In other words, *all* chapters within a state must use the round officially in order for *any* chapter within a state to use it officially. All students, whether registered as part of a school team or as individual competitors, are eligible to qualify for the Countdown Round.

An official Countdown Round determines an individual's final overall rank in the competition. If a Countdown Round is used officially, the official procedures as established by the MATHCOUNTS Foundation must be followed, as described below.*

- The top 25% of students, up to a maximum of 10, are selected to compete. These students are chosen based on their Individual Scores.
- The two lowest-ranked students are paired; a question is read and projected, and students are given 45 seconds to solve the problem. A student may buzz in at any time, and if s/he answers correctly, a point is scored. If a student answers incorrectly, the other student has the remainder of the 45 seconds to answer.
- Three total questions are read to the pair of students, one question at a time, and the student who scores the higher number of points (not necessarily 2 out of 3) progresses to the next round and challenges the next-higher-ranked student.
- If students are tied in their matchup after three questions (at 1-1 or 0-0), questions should continue to be read until one is successfully answered. The first student who answers an additional question correctly progresses to the next round.
- This procedure continues until the 4th-ranked Mathlete and his/her opponent compete. For the final four matchups, the first student to correctly answer three questions advances.
- The Countdown Round proceeds until a 1st place individual is identified. More details about Countdown Round procedures are included in the 2016 School Competition.

**Rules for the Countdown Round change for the National Competition.*

An unofficial Countdown Round does not determine an individual's final overall rank in the competition, but is done for practice or for fun. The official procedures do not have to be followed. Chapters and states choosing not to conduct the round officially must determine individual winners solely on the basis of students' scores in the Target and Sprint Rounds of the competition.

SCORING

MATHCOUNTS Competition Series scores do not conform to traditional grading scales. Coaches and students should view an Individual Score of 23 (out of a possible 46) as highly commendable.

INDIVIDUAL SCORE: calculated by taking the sum of the number of Sprint Round questions answered correctly and twice the number of Target Round questions answered correctly. There are 30 questions in the Sprint Round and eight questions in the Target Round, so the maximum possible Individual Score is $30 + 2(8) = 46$. If used officially, the Countdown Round yields final individual standings.

TEAM SCORE: calculated by dividing the sum of the team members' Individual Scores by four (even if the team has fewer than four members) and adding twice the number of Team Round questions answered correctly. The highest possible Individual Score is 46. Four students may compete on a team, and there are 10 questions in the Team Round. Therefore, the maximum possible Team Score is $((46 + 46 + 46 + 46) \div 4) + 2(10) = 66$.

TIEBREAKING ALGORITHM: used to determine team and individual ranks and to determine which individuals qualify for the Countdown Round. In general, questions in the Target, Sprint and Team Rounds increase in difficulty so that the most difficult questions occur near the end of each round. In a comparison of questions to break ties, generally those who correctly answer the more difficult questions receive the higher rank. The guidelines provided below are very general; competition officials receive more detailed procedures.

- *Ties between individuals:* the student with the higher Sprint Round score will receive the higher rank. If a tie remains after this comparison, specific groups of questions from the Target and Sprint Rounds are compared.
- *Ties between teams:* the team with the higher Team Round score, and then the higher sum of the team members' Sprint Round scores, receives the higher rank. If a tie remains after these comparisons, specific questions from the Team Round will be compared.

RESULTS DISTRIBUTION

Coaches should expect to receive the scores of their students, as well as a list of the top 25% of students and top 40% of teams, from their competition coordinators. In addition, single copies of the blank competition materials and answer keys may be distributed to coaches after all competitions at that level nationwide have been completed. *Before distributing blank competition materials and answer keys, coordinators must wait for verification from the national office that all such competitions have been completed.* Both the problems and answers from Chapter and State Competitions will be posted on the MATHCOUNTS website following the completion of all competitions at that level nationwide, replacing the previous year's posted tests.

Student competition papers and answers will not be viewed by or distributed to coaches, parents, students or other individuals. Students' competition papers become the confidential property of MATHCOUNTS.

ADDITIONAL RULES

All answers must be legible.

Pencils and paper will be provided for Mathletes by competition organizers. However, students may bring their own pencils, pens and erasers if they wish. They may not use their own scratch paper or graph paper.

Use of notes or other reference materials (including dictionaries and translation dictionaries) is prohibited.

Specific instructions stated in a given problem take precedence over any general rule or procedure.

Communication with coaches is prohibited during rounds but is permitted during breaks. All communication between guests and Mathletes is prohibited during competition rounds. Communication between teammates is permitted only during the Team Round.

Calculators are not permitted in the Sprint and Countdown Rounds, but they are permitted in the Target, Team and Tiebreaker (if needed) Rounds. When calculators are permitted, students may use any calculator (including programmable and graphing calculators) that does not contain a QWERTY (typewriter-like) keypad. Calculators that have the ability to enter letters of the alphabet but do not have a keypad in a standard typewriter arrangement are acceptable. Smart phones, laptops, tablets, iPods®, personal digital assistants (PDAs) and any other “smart” devices are not considered to be calculators and may not be used during competitions. Students may not use calculators to exchange information with another person or device during the competition.

Coaches are responsible for ensuring their students use acceptable calculators, and students are responsible for providing their own calculators. Coordinators are not responsible for providing Mathletes with calculators or batteries before or during MATHCOUNTS competitions. Coaches are strongly advised to bring backup calculators and spare batteries to the competition for their team members in case of a malfunctioning calculator or weak or dead batteries. Neither the MATHCOUNTS Foundation nor coordinators shall be responsible for the consequences of a calculator’s malfunctioning.

Pagers, cell phones, tablets, iPods® and other MP3 players should not be brought into the competition room. Failure to comply could result in dismissal from the competition.

Should there be a rule violation or suspicion of irregularities, the MATHCOUNTS coordinator or competition official has the obligation and authority to exercise his/her judgment regarding the situation and take appropriate action, which might include disqualification of the suspected student(s) from the competition.

FORMS OF ANSWERS

The following rules explain acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Competition answers will be scored in compliance with these rules for forms of answers.

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lb 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, \$0.25 will not be accepted.

All answers must be expressed in simplest form. A "common fraction" is to be considered a fraction in the form $\pm\frac{a}{b}$, where a and b are natural numbers and GCF(a, b) = 1. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not have a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N, a and b are natural numbers, $a < b$ and GCF(a, b) = 1. Examples:

Problem: What is 8 ÷ 12 expressed as a common fraction? *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$

Problem: What is 12 ÷ 8 expressed as a common fraction? *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$

Problem: What is the sum of the lengths of the radius and the circumference of a circle of diameter $\frac{1}{4}$ unit expressed as a common fraction in terms of π? *Answer:* $\frac{1+2\pi}{8}$

Problem: What is 20 ÷ 12 expressed as a mixed number? *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:

Acceptable Simplified Forms: $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}$, $\frac{\frac{1}{4}}{3}$, 3.5, 2:1

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:
Problem: What is $\sqrt{15} \times \sqrt{5}$ expressed in simplest radical form? *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form (\$) *a.bc*, where *a* is an integer and *b* and *c* are digits. The *only* exceptions to this rule are when *a* is zero, in which case it may be omitted, or when *b* and *c* are both zero, in which case they both may be omitted. Answers in the form (\$) *a.bc* should be rounded to the nearest cent, unless otherwise specified. Examples:
Acceptable Forms: 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

Do not make approximations for numbers (e.g., π, $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not do any intermediate rounding (other than the "rounding" a calculator performs) when calculating solutions. All rounding should be done at the end of the calculation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \leq |a| < 10$, and n is an integer. Examples:
Problem: What is 6895 expressed in scientific notation? *Answer:* 6.895×10^3
Problem: What is 40,000 expressed in scientific notation? *Answer:* 4×10^4 *or* 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole-number answers should be expressed in their whole-number form. Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.

VOCABULARY AND FORMULAS

The following list is representative of terminology used in the problems but should not be viewed as all-inclusive. It is recommended that coaches review this list with their Mathletes.

absolute difference
absolute value
acute angle
additive inverse *(opposite)*
adjacent angles
algorithm
alternate exterior angles
alternate interior angles
altitude *(height)*
apex
area
arithmetic mean
arithmetic sequence
base 10
binary
bisect
box-and-whisker plot
center
chord
circle
circumference
circumscribe
coefficient
collinear
combination
common denominator
common divisor
common factor
common fraction
common multiple
complementary angles
composite number
compound interest
concentric
cone
congruent
convex
coordinate plane/system
coordinates of a point
coplanar
corresponding angles
counting numbers
counting principle
cube
cylinder
decagon
decimal
degree measure
denominator
diagonal of a polygon
diagonal of a polyhedron
diameter
difference
digit
digit-sum
direct variation
dividend
divisible
divisor
dodecagon
dodecahedron
domain of a function
edge
endpoint
equation
equiangular
equidistant
equilateral
evaluate
expected value
exponent
expression
exterior angle of a polygon
factor
factorial
finite
formula
frequency distribution
frustum
function
GCF
geometric mean
geometric sequence
height *(altitude)*
hemisphere
heptagon
hexagon
hypotenuse
image(s) of a point(s) *(under a transformation)*
improper fraction
inequality
infinite series
inscribe
integer
interior angle of a polygon
interquartile range
intersection
inverse variation
irrational number
isosceles
kite
lateral edge
lateral surface area
lattice point(s)
LCM
linear equation
mean
median of a set of data
median of a triangle
midpoint
mixed number
mode(s) of a set of data
multiple
multiplicative inverse *(reciprocal)*
natural number
nonagon
numerator
obtuse angle
octagon
octahedron
ordered pair
origin
palindrome
parallel
parallelogram
Pascal's Triangle
pentagon
percent increase/decrease
perimeter
permutation
perpendicular
planar
polygon
polyhedron
polynomial
prime factorization
prime number

principal square root
prism
probability
product
proper divisor
proper factor
proper fraction
proportion
pyramid
Pythagorean Triple
quadrant
quadrilateral
quotient
radius
random
range of a data set
range of a function
rate
ratio
rational number
ray
real number
reciprocal *(multiplicative inverse)*
rectangle
reflection
regular polygon
relatively prime
remainder
repeating decimal
revolution
rhombus
right angle
right circular cone
right circular cylinder
right polyhedron
right triangle
rotation
scalene triangle
scientific notation
sector
segment of a circle
segment of a line
semicircle
semiperimeter
sequence
set
significant digits
similar figures
simple interest
slope
slope-intercept form
solution set
space diagonal
sphere
square
square root
stem-and-leaf plot
sum
supplementary angles
system of equations/inequalities
tangent figures
tangent line
term
terminating decimal
tetrahedron
total surface area
transformation
translation
trapezoid
triangle
triangular numbers
trisect
twin primes
union
unit fraction
variable
vertex
vertical angles
volume
whole number
x-axis
x-coordinate
x-intercept
y-axis
y-coordinate
y-intercept

The list of formulas below is representative of those needed to solve MATHCOUNTS problems but should not be viewed as the only formulas that may be used. Many other formulas that are useful in problem solving should be discovered and derived by Mathletes.

CIRCUMFERENCE

Circle	$C = 2 \times \pi \times r = \pi \times d$

AREA

Circle	$A = \pi \times r^2$
Square	$A = s^2$
Rectangle	$A = l \times w = b \times h$
Parallelogram	$A = b \times h$
Trapezoid	$A = \frac{1}{2}(b_1 + b_2) \times h$
Rhombus	$A = \frac{1}{2} \times d_1 \times d_2$
Triangle	$A = \frac{1}{2} \times b \times h$
Triangle	$A = \sqrt{s(s-a)(s-b)(s-c)}$
Equilateral triangle	$A = \frac{s^2\sqrt{3}}{4}$

SURFACE AREA AND VOLUME

Sphere	$SA = 4 \times \pi \times r^2$
Sphere	$V = \frac{4}{3} \times \pi \times r^3$
Rectangular prism	$V = l \times w \times h$
Circular cylinder	$V = \pi \times r^2 \times h$
Circular cone	$V = \frac{1}{3} \times \pi \times r^2 \times h$
Pyramid	$V = \frac{1}{3} \times B \times h$

Pythagorean Theorem	$c^2 = a^2 + b^2$
Counting/ Combinations	${}_nC_r = \frac{n!}{r!(n-r)!}$

ANSWERS

In addition to the answer, we have provided a difficulty rating for each problem. Our scale is 1-7, with 7 being the most difficult. These are only approximations, and how difficult a problem is for a particular student will vary. Below is a general guide to the ratings:

Difficulty 1/2/3 - One concept; one- to two-step solution; appropriate for students just starting the middle school curriculum.
4/5 - One or two concepts; multistep solution; knowledge of some middle school topics is necessary.
6/7 - Multiple and/or advanced concepts; multistep solution; knowledge of advanced middle school topics and/or problem-solving strategies is necessary.

Warm-Up 1

Answer		Difficulty			
1.	200	(1)	6.	88	(2)
2.	59	(2)	7.	7	(2)
3.	1080	(3)	8.	277	(2)
4.	31.68	(2)	9.	13	(3)
5.	8	(3)	10.	−800	(4)

Warm-Up 2

Answer		Difficulty			
11.	1	(1)	16.	20	(5)
12.	2	(2)	17.	6	(2)
13.	12	(2)	18.	113	(3)
14.	5/3	(3)	19.	225	(4)
15.	36,300	(3)	20.	16/47	(4)

Warm-Up 3

Answer		Difficulty			
21.	4	(2)	26.	81	(4)
22.	223,200	(2)	27.	7.5	(4)
23.	1/6	(4)	28.	9	(3)
24.	20	(4)	29.	107	(5)
25.	1/3	(5)	30.	10	(4)

Workout 1

Answer		Difficulty			
31.	3.6	(2)	36.	7.2	(3)
32.	28	(2)	37.	32.6	(4)
33.	64.5	(3)	38.	5	(4)
34.	1.45	(5)	39.	1×10^9	(4)
35.	13	(2)	40.	12	(4)

Workout 2

Answer		Difficulty			
41.	6	(3)	46.	16.4	(3)
42.	19.2	(4)	47.	0.8	(2)
43.	5 or 5.00	(3)	48.	40	(3)
44.	42.9	(4)	49.	72	(4)
45.	2.70	(4)	50.	1*	(3)

Warm-Up 4

Answer		Difficulty			
51.	50	(2)	56.	5940	(3)
52.	$18\frac{1}{3}$	(2)	57.	55	(4)
53.	red	(2)	58.	48	(4)
54.	8	(5)	59.	550	(5)
55.	259	(4)	60.	20	(4)

** The plural form of the units is always provided in the answer blank, even if the answer appears to require the singular form of the units.*

Warm-Up 5

Answer	Difficulty			
61. 34	(4)	66.	15	(3)
62. 24	(3)	67.	1/5	(4)
63. 25	(3)	68.	Monday	(3)
64. 75 or 75.00	(3)	69.	6	(4)
65. 432	(4)	70.	15	(4)

Workout 4

Answer	Difficulty			
91. 15	(3)	96.	1091	(5)
92. 0.9	(4)	97.	3	(3)
93. 7.5	(4)	98.	30.8	(3)
94. 3.04	(4)	99.	55	(2)
95. 157.5	(4)	100.	7	(4)

Warm-Up 6

Answer	Difficulty			
71. 2	(4)	76.	−27	(4)
72. 15	(3)	77.	$328\frac{1}{8}$	(4)
73. 60	(3)	78.	3	(4)
74. 64,000	(5)	79.	511/512	(4)
75. 88	(3)	80.	60 or 60.00	(3)

Warm-Up 7

Answer	Difficulty			
101. 199	(3)	106.	17/14	(3)
102. 14	(3)	107.	25	(6)
103. 1.25	(3)	108.	226	(3)
104. 40	(4)	109.	64	(3)
105. $26\frac{5}{8}$	(3)	110.	10	(3)

Workout 3

Answer	Difficulty			
81. 25	(3)	86.	12	(5)
82. 4.05	(3)	87.	96	(5)
83. 240	(4)	88.	6.83	(4)
84. 5.6	(4)	89.	1.75	(5)
85. 0.025	(3)	90.	24	(3)

Warm-Up 8

Answer	Difficulty			
111. 121	(3)	116.	−144	(4)
112. 12	(4)	117.	3	(3)
113. 6	(4)	118.	14	(5)
114. 5850	(4)	119.	5	(4)
115. 5/41	(4)	120.	9	(5)

Workout 5

Answer	Difficulty		
121. 4.7	(3)	126. 26	(4)
122. −68.75	(4)	127. 46,656	(5)
123. 8.6	(4)	128. 4	(5)
124. 0.79	(5)	129. 11/10	(3)
125. 65	(3)	130. 25	(4)

Workout 6

Answer	Difficulty		
131. 20	(3)	136. 8.4	(5)
132. 6/25	(4)	137. 48	(5)
133. $2 - \sqrt{3}$ or $-\sqrt{3} + 2$	(5)	138. 0.84	(2)
134. 11.1	(5)	139. 5.7	(5)
135. $\sqrt{3}$	(5)	140. 6	(3)

Warm-Up 9

Answer	Difficulty		
141. 19	(4)	146. 12	(5)
142. 1008	(4)	147. 23	(4)
143. 45	(3)	148. 15	(4)
144. 10	(4)	149. 23	(4)
145. 78	(5)	150. 22	(4)

Warm-Up 10

Answer	Difficulty		
151. 7	(4)	156. 17	(3)
152. 50	(3)	157. 29,400	(4)
153. 47	(4)	158. 36	(5)
154. 29	(4)	159. 17	(3)
155. 20	(5)	160. $\sqrt{145}$	(4)

Warm-Up 11

Answer	Difficulty		
161. 60	(4)	166. 7	(5)
162. 45	(3)	167. $2\pi + 12$ or $12 + 2\pi$	(5)
163. 36	(5)	168. 3	(4)
164. 625	(5)	169. 16	(4)
165. 2016	(5)	170. 90	(4)

Workout 7

Answer	Difficulty		
171. −5.6	(5)	176. 6	(5)
172. 19.80	(4)	177. $10\pi - 4$ or $-4 + 10\pi$	(5)
173. 55.2	(5)	178. 678	(4)
174. 0.17	(5)	179. 9	(5)
175. 2014	(5)	180. 5/12	(5)

Workout 8

Answer	Difficulty		
181. 36	(3)	186. 9.2	(5)
182. 2048	(4)	187. 93	(4)
183. $22\sqrt{6}$	(6)	188. 66	(4)
184. $\sqrt{38}$	(5)	189. 1275	(4)
185. 547	(4)	190. 145	(6)

Warm-Up 12

Answer	Difficulty		
191. 1/24	(4)	196. 5	(5)
192. 20	(3)	197. 7 or 7.00	(3)
193. 4/3	(5)	198. 54	(5)
194. 135	(4)	199. 1/7	(4)
195. 33/20	(4)	200. −3	(4)

Warm-Up 13

Answer	Difficulty		
201. $9\frac{1}{3}$	(3)	206. 243	(5)
202. 24	(3)	207. 3/7	(3)
203. 16	(4)	208. 126	(5)
204. 1/2	(4)	209. (6, 2)	(5)
205. 192.5	(5)	210. 875/64	(6)

Warm-Up 14

Answer	Difficulty		
211. 20	(4)	216. 90	(5)
212. 191/20	(4)	217. 1	(4)
213. 157	(4)	218. 13/34	(4)
214. 64	(3)	219. 11	(4)
215. 31	(4)	220. 124	(5)

Counting Stretch

Answer	Difficulty		
221. 855	(2)	226. 10	(3)
222. 56	(3)	227. 14	(3)
223. 27	(3)	228. 100	(3)
224. 87	(3)	229. 64	(4)
225. 13	(4)	230. 54	(4)

Area Stretch

Answer	Difficulty		
231. 92	(3)	236. $4 + 8\pi$ or $8\pi + 4$	(5)
232. 321	(2)	237. $128 + 32\sqrt{2}$ or $32\sqrt{2} + 128$	(6)
233. 7000	(3)	238. 357/4	(6)
234. 32	(3)	239. 9/4	(7)
235. $4\pi - 8$ or $-8 + 4\pi$	(4)	240. 240/37	(6)

Modular Arithmetic Stretch

Answer	Difficulty		
241. March	(3)	246. 2	(4)
242. 11:00 p.m.	(3)	247. 3	(4)
243. 110	(4)	248. 83	(5)
244. 8	(5)	249. 61	(6)
245. 5	(4)	250. 59	(6)

MATHCOUNTS Problems Mapped to Common Core State Standards (CCSS)

Forty-three states, the District of Columbia, four territories and the Department of Defense Education Activity (DoDEA) have adopted the Common Core State Standards (CCSS). As such, MATHCOUNTS considers it beneficial for teachers to see the connections between the *2015-2016 MATHCOUNTS School Handbook* problems and the CCSS. MATHCOUNTS not only has identified a general topic and assigned a difficulty level for each problem but also has provided a CCSS code in the Problem Index (pages 62-63). A complete list of the Common Core State Standards can be found at www.corestandards.org.

The CCSS for mathematics cover K-8 and high school courses. MATHCOUNTS problems are written to align with the NCTM Standards for Grades 6-8. As one would expect, there is great overlap between the two sets of standards. MATHCOUNTS also recognizes that in many school districts, algebra and geometry are taught in middle school, so some MATHCOUNTS problems also require skills taught in those courses.

In referring to the CCSS, the Problem Index code for each of the Standards for Mathematical Content for grades K-8 begins with the grade level. For the Standards for Mathematical Content for high school courses (such as algebra or geometry), each code begins with a letter to indicate the course name. The second part of each code indicates the domain within the grade level or course. Finally, the number of the individual standard within that domain follows. Here are two examples:

- *6.RP.3 → Standard #3 in the Ratios and Proportional Relationships domain of grade 6*
- *G-SRT.6 → Standard #6 in the Similarity, Right Triangles and Trigonometry domain of Geometry*

Some math concepts utilized in MATHCOUNTS problems are not specifically mentioned in the CCSS. Two examples are the Fundamental Counting Principle (FCP) and special right triangles. In cases like these, if a related standard could be identified, a code for that standard was used. For example, problems using the FCP were coded 7.SP.8, S-CP.8 or S-CP.9 depending on the context of the problem; SP → Statistics and Probability (the domain), S → Statistics and Probability (the course) and CP → Conditional Probability and the Rules of Probability. Problems based on special right triangles were given the code G-SRT.5 or G-SRT.6, explained above.

There are some MATHCOUNTS problems that either are based on math concepts outside the scope of the CCSS or based on concepts in the standards for grades K-5 but are obviously more difficult than a grade K-5 problem. When appropriate, these problems were given the code SMP for Standards for Mathematical Practice. The CCSS include the Standards for Mathematical Practice along with the Standards for Mathematical Content. The SMPs are (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (4) Model with mathematics; (5) Use appropriate tools strategically; (6) Attend to precision; (7) Look for and make use of structure and (8) Look for and express regularity in repeated reasoning.

PROBLEM INDEX

It is difficult to categorize many of the problems in the *MATHCOUNTS School Handbook*. It is very common for a MATHCOUNTS problem to straddle multiple categories and cover several concepts. This index is intended to be a helpful resource, but since each problem has been placed in exactly one category and mapped to exactly one Common Core State Standard (CCSS), the index is not perfect. In this index, the code **9 (3) 7.SP.3** refers to problem 9 with difficulty rating 3 mapped to CCSS 7.SP.3. For an explanation of the difficulty ratings refer to page 57. For an explanation of the CCSS codes refer to page 61.

Proportional Reasoning

4	(2)	6.RP.3
18	(3)	6.RP.3
36	(3)	6.NS.1
42	(4)	6.RP.3
56	(3)	7.NS.3
60	(4)	7.RP.3
94	(4)	6.EE.7
103	(3)	6.RP.3
106	(3)	6.RP.3
109	(3)	6.RP.3
192	(3)	6.RP.3
204	(4)	7.RP.2

Number Theory

9	(3)	4.OA.4
32	(2)	7.NS.2
39	(4)	8.EE.2
51	(2)	7.NS.3
64	(4)	7.NS.3
68	(3)	7.NS.3
69	(4)	SMP
81	(3)	F-BF.2
93	(4)	SMP
113	(4)	SMP
119	(3)	6.NS.3
129	(3)	7.NS.3
130	(4)	SMP
131	(3)	SMP
144	(4)	SMP
165	(5)	7.NS.2
166	(5)	8.EE.1
185	(4)	SMP
213	(4)	SMP
218	(4)	7.SP.7
220	(5)	SMP

Modular Arithmetic Stretch*

Algebraic Expressions & Equations

8	(2)	6.EE.3
10	(4)	6.EE.7
23	(4)	8.EE.7
34	(5)	F-IF.2
35	(2)	A-REI.4
38	(4)	8.F.3
43	(3)	A-CED.2
57	(4)	8.EE.8
61	(4)	8.EE.8
71	(4)	A-CED.1
74	(4)	A-SSE.3
86	(5)	A-CED.2
99	(1)	F-IF.2
108	(3)	6.EE.2
112	(4)	6.EE.2
122	(4)	8.EE.8
125	(3)	F-IF.2
128	(5)	8.EE.8
134	(5)	8.EE.7
140	(3)	F-IF.2
141	(4)	8.F.1
147	(4)	A-SSE.3
148	(4)	4.OA.4
168	(4)	A-SSE.3
171	(5)	8.F.3
182	(4)	F-IF.2
195	(4)	8.F.3
196	(5)	A-SSE.2
200	(4)	A-SSE.3
203	(5)	F-IF.2
206	(3)	A-SSE.1
211	(4)	N-RN.2

Problem Solving (Misc.)

7	(2)	SMP
11	(1)	SMP
12	(2)	SMP
48	(3)	SMP
66	(3)	SMP
101	(3)	SMP
116	(4)	SMP
126	(4)	SMP
150	(4)	SMP
156	(3)	SMP
159	(3)	SMP
162	(3)	SMP
187	(4)	S-CP.9
188	(4)	SMP
189	(4)	S-CP.9
190	(6)	SMP
197	(3)	SMP
198	(5)	SMP
207	(3)	SMP
216	(5)	SMP

General Math

1	(1)	4.OA.2
2	(2)	7.NS.3
14	(3)	6.NS.1
22	(2)	4.OA.2
73	(3)	4.OA.2
80	(3)	N-Q.1
82	(3)	7.NS.3
91	(3)	A-CED.1
105	(3)	7.NS.3
110	(3)	SMP
111	(3)	7.NS.3
138	(2)	6.NS.3
155	(5)	SMP
202	(3)	SMP

Plane Geometry

3	(3)	7.G.5
13	(2)	7.G.4
15	(3)	7.G.6
19	(4)	G-C.2
40	(4)	7.G.6
41	(3)	SMP
47	(2)	G-C.2
52	(2)	6.NS.1
58	(4)	7.G.6
70	(4)	8.G.8
72	(3)	7.G.5
85	(3)	7.G.4
87	(5)	7.G.6
88	(4)	7.NS.3
96	(5)	8.G.7
98	(3)	7.G.4
102	(3)	8.G.8
107	(6)	8.G.5
120	(3)	G-SRT.5
123	(4)	G-C.2
133	(5)	G-SRT.6
137	(5)	7.G.5
139	(5)	G-SRT.6
146	(5)	G-SRT.6
160	(4)	8.G.7
161	(4)	A-SSE.3
164	(5)	8.G.7
167	(5)	G-SRT.6
174	(5)	G-SRT.6
178	(4)	G-SRT.6
183	(6)	7.G.6
184	(5)	8.G.7

Area Stretch*

Sequences, Series & Patterns

17	(2)	F-BF.2
24	(4)	F-BF.2
30	(4)	F-BF.2
76	(4)	F-BF.2
79	(4)	F-BF.2
114	(4)	F-LE.2
142	(4)	F-BF.2
153	(4)	F-LE.2
163	(5)	F-LE.2
175	(5)	F-BF.2
179	(5)	F-BF.2
215	(4)	F-BF.2

Solid Geometry

28	(3)	G-GMD.3
45	(4)	G-GMD.3
50	(3)	G-GMD.3
63	(5)	7.G.6
65	(5)	7.G.6
89	(5)	G-GMD.3
90	(3)	8.G.9
92	(4)	8.G.9
124	(5)	G-GMD.3
186	(5)	G-GMD.3
205	(4)	G-GMD.4
214	(3)	G-GMD.3

Coordinate Geometry

154	(4)	6.G.3
158	(5)	A-SSE.3
177	(5)	7.G.6
209	(5)	8.G.1
210	(6)	8.G.8
217	(4)	8.G.8

Measurement

29	(5)	6.RP.3
31	(2)	6.RP.3
37	(4)	7.RP.3
59	(5)	7.RP.3
75	(3)	7.G.6
95	(4)	6.RP.3
143	(3)	6.RP.3
170	(4)	7.RP.3
194	(4)	6.RP.3
201	(3)	7.RP.3
219	(4)	6.RP.3

Statistics

6	(2)	6.SP.5
33	(3)	6.SP.5
54	(5)	F-BF.2
104	(4)	6.SP.5
118	(4)	6.SP.5
149	(4)	6.SP.5

Logic

21	(2)	8.G.3
53	(2)	SMP
78	(4)	SMP
97	(3)	SMP
135	(5)	G-SRT.6
152	(3)	SMP
169	(4)	SMP

Percents & Fractions

16	(5)	6.EE.7
27	(4)	7.RP.3
46	(3)	7.NS.3
77	(4)	7.NS.3
83	(4)	6.RP.3
100	(4)	7.NS.3
121	(3)	6.RP.3
136	(5)	7.RP.3
172	(4)	7.RP.3
176	(5)	6.RP.3
193	(5)	A-CED.2
212	(4)	7.NS.3

Probability, Counting & Combinatorics

5	(3)	S-CP.9
20	(4)	7.SP.8
25	(5)	7.SP.3
26	(4)	S-CP.9
44	(4)	7.SP.8
49	(4)	S-CP.9
55	(4)	7.SP.8
62	(3)	S-CP.9
67	(4)	7.SP.8
84	(4)	7.SP.7
115	(4)	7.SP.8
117	(3)	7.SP.7
127	(5)	S-CP.9
132	(4)	7.SP.8
145	(5)	S-CP.9
151	(4)	S-CP.9
157	(4)	S-CP.9
173	(5)	7.SP.8
180	(5)	7.SP.8
181	(3)	4.OA.4
191	(4)	7.SP.8
199	(4)	7.SP.8
208	(5)	S-CP.9

Counting Stretch*

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2014-2015 QUESTION WRITING COMMITTEE

Chair: Edward Early, St. Edward's University, *Austin, TX*
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Sandra Powers, *Daniel Island, SC*
Randy Rogers (NAT 85), *Davenport, IA*
Dianna Sopala, *Fair Lawn, NJ*
Craig Volden (NAT 84), *Earlysville, VA*

Special Thanks to:

Mady Bauer, *Bethel Park, PA*
Jerrold W. Grossman, *Rochester, MI*
Jane Lataille, *Los Alamos, NM*
Leon Manelis, *Orlando, FL*

The **Solutions** to the problems were written by Kent Findell, Diamond Middle School, *Lexington, MA.*

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MATHCOUNTS FOUNDATION

Editor and Contributing Author: Kera Johnson, *Manager of Education*

Content Editor: Cara Norton, *Senior Education Coordinator*

Author of Introduction & Program Information: Amanda Naar, *Communications Manager*

Executive Director: Louis DiGioia

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1420 King Street, Alexandria, VA 22314
703-299-9006 ♦ www.mathcounts.org ♦ info@mathcounts.org