
MATHCOUNTS®

2012

■ Chapter Competition ■

Sprint Round

Problems 1–30

HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature _____ Date _____

Printed Name _____

School _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

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1. _____ hours



Mrs. Smith teaches for $5\frac{1}{2}$ hours each day. If she teaches for six days, for how many total hours does Mrs. Smith teach?

2. _____

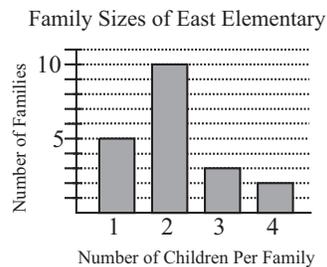
An operation \otimes is defined as $a \otimes b = 6a - b$. What is the value of $4 \otimes 22$?

3. _____ dollars

If 7 apples cost x dollars, how many dollars do 70 apples cost? Express your answer in terms of x .



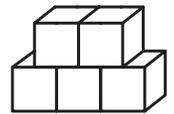
4. _____ families



The results of a survey of the 20 families at East Elementary are shown in the graph. During the survey, each family was asked how many children were in their family. How many of the families surveyed had three or more children?

5. _____ ounces

The total weight of five identical blocks is 3 pounds 12 ounces. There are 16 ounces in a pound. How many ounces does each block weigh?



6. _____

What is the value of x if $\frac{4}{2x} = \frac{1}{3}$?

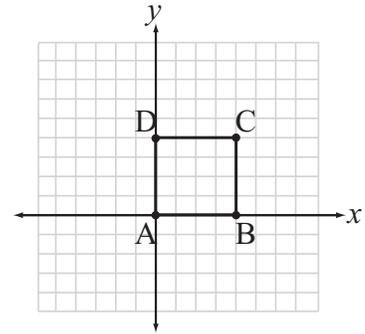
7. _____ cups



Maureen rented a lemonade stand for \$10.00 per week. She sells her lemonade for 75¢ per cup, but also spends 25¢ per cup of lemonade on supplies. How many cups of lemonade must she sell in one week so that her costs (rent and supplies) are equal to her sales?

8. _____ cents Hugo has 5 quarters, 13 dimes and 7 nickels. Luigi has 6 quarters, 10 dimes and 2 nickels. In cents, how much more than Luigi does Hugo have?

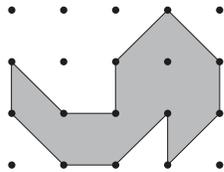
9. _____ (,) Square ABCD has vertices $A(0,0)$, $B(4,0)$, $C(4,4)$ and $D(0,4)$. A point P in the interior of square ABCD is chosen and reflected over the line containing segment CD to point Q . Point Q is then reflected over the line containing segment BC to point R . What are the coordinates of the midpoint of segment PR ? Express your answer as an ordered pair.



10. _____ % What percent of the positive integers less than or equal to 36 are factors of 36?

11. _____ dollars In one round of a television game show, three questions are asked. The second question is worth twice as much as the first. The third question is worth three times as much as the second. If the third question is worth \$12,000, what is the first question worth, in dollars?

12. _____ cm^2



In the figure shown, the distance between adjacent dots in each row and in each column is 1 cm. In square centimeters, what is the area of the shaded region?

13. _____ Seven consecutive positive integers have a sum of 91. What is the largest of these integers?

14. _____ If Friday the 13th occurs in a month, then the sum of the calendar dates for the Fridays in that month is $6 + 13 + 20 + 27 = 66$. What is the largest sum of calendar dates for seven consecutive Fridays in any given year?

15. _____ cm^3 A cylindrical container has a diameter of 8 cm and a volume of 754 cm^3 . A different cylindrical container, which also has a diameter of 8 cm, is twice as tall as the original container. What is the volume of the second container?

16. _____ degrees The angles of a triangle are in the ratio 1:3:5. What is the degree measure of the largest angle in the triangle?

17. _____ Four numbers are written in a row. The mean of the first two numbers is 10, and the mean of the last two numbers is 20. What is the mean of all four numbers?

18. _____ seconds



Carol, Jane, Kim, Nancy and Vicky, competed in a 400-meter race.

- Nancy beat Jane by 6 seconds.
 - Carol finished 11 seconds behind Vicky.
 - Nancy finished 2 seconds ahead of Kim, but 3 seconds behind Vicky.
- By how many seconds did Kim finish the race ahead of Carol?

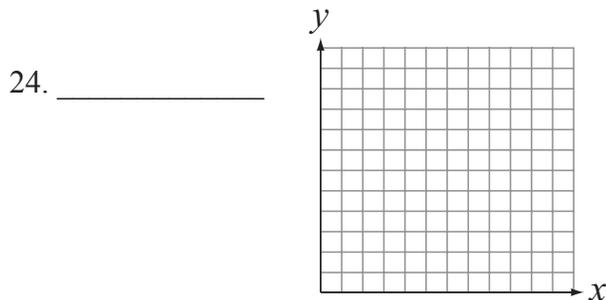
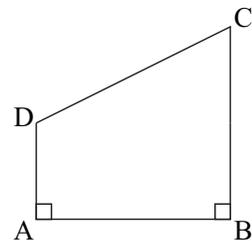
19. _____ S and T are both two-digit integers less than 80. Each number is divisible by 3, and T also is divisible by 7. S is a perfect square. If the sum $S + T$ is a multiple of 11, what is the value of T ?

20. _____ In a stack of six cards, each card is labeled with a different integer from 0 to 5, inclusive. If two cards are selected at random without replacement, what is the probability that their sum will be 3? Express your answer as a common fraction.

21. _____ minutes Okta can stay in the sun for 16 minutes before getting sunburned. If he uses a sunscreen with SPF 20, he can stay in the sun 20 times as long before getting sunburned, or $20 \times 16 = 320$ minutes. This morning, he spent 9 minutes in the sun before putting on sunscreen. If he then applies SPF 20 sunscreen, how many more minutes can he stay in the sun before getting burned?

22. _____ A bag contains red and yellow balls. There are ten of each color and they are numbered 1 through 10. If Arthur draws two balls at random, without replacement, what is the probability that he draws the yellow ball numbered 3 followed by a red ball? Express your answer as a common fraction.

23. _____ units² In trapezoid ABCD, $AB = BC = 2AD$ and $AD = 5$ units. What is the area of trapezoid ABCD?

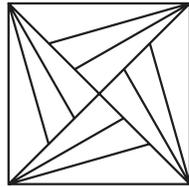


One line has a slope of $-\frac{1}{3}$ and contains the point $(3, 6)$. Another line has a slope of $\frac{5}{3}$ and contains the point $(3, 0)$. What is the product of the coordinates of the point at which the two lines intersect?

25. _____ miles A car traveled a certain distance at 20 miles per hour. The car then traveled twice the original distance at 40 miles per hour. If the entire trip lasted four hours, what was the total number of miles driven?

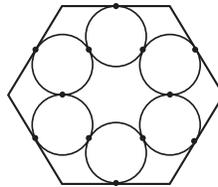
26. _____ Consider all integer values of a and b for which $a < 2$ and $b \geq -2$. What is the minimum value of $b - a$?

27. _____ triangles



In the figure shown, the diagonals of a square are drawn and then two additional segments are drawn from each vertex to a diagonal. How many triangles are in the figure?

28. _____ units Six circles of radius 1 unit are drawn tangent to the sides of a regular hexagon. Each circle is also tangent to two other circles as shown in the drawing. What is the perimeter of the hexagon? Express your answer in simplest radical form.



29. _____ If $x^2 + \frac{1}{x^2} = 3$, what is the value of $\frac{x^2}{(x^2 + 1)^2}$? Express your answer as a common fraction.

30. _____ cm A circle is inscribed in a rhombus with sides of length 4 cm. If the two acute angles in the rhombus each measure 60° , what is the length of the circle's radius? Express your answer in simplest radical form.



Forms of Answers

The following list explains acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Judges will score competition answers in compliance with these rules for forms of answers.

All answers must be expressed in simplest form. A “common fraction” is to be considered a fraction in the form $\pm \frac{a}{b}$, where a and b are natural numbers and $\text{GCF}(a, b) = 1$. In some cases the term “common fraction” is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not share a common factor. A simplified “mixed number” (“mixed numeral,” “mixed fraction”) is to be considered a fraction in the form $\pm N \frac{a}{b}$, where N , a and b are natural numbers, $a < b$ and $\text{GCF}(a, b) = 1$. Examples:

Problem: Express 8 divided by 12 as a common fraction. *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$
Problem: Express 12 divided by 8 as a common fraction. *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}, 1\frac{1}{2}$
Problem: Express the sum of the lengths of the radius and the circumference of a circle with a diameter of $\frac{1}{4}$ as a common fraction in terms of π . *Answer:* $\frac{1+2\pi}{8}$
Problem: Express 20 divided by 12 as a mixed number. *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}, \frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:

Simplified, Acceptable Forms: $\frac{7}{2}, \frac{3}{\pi}, \frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}, \frac{1}{3}, 3.5, 2:1$

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:

Problem: Evaluate $\sqrt{15} \times \sqrt{5}$. *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., “How many dollars...,” “How much will it cost...,” “What is the amount of interest...”) should be expressed in the form (\$) $a.bc$, where a is an integer and b and c are digits.

The *only* exceptions to this rule are when a is zero, in which case it may be omitted, or when b and c are both zero, in which case they may both be omitted. Examples:

Acceptable: 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lbs 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, \$0.25 will not be accepted.

Do not make approximations for numbers (e.g., π , $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not do any intermediate rounding (other than the “rounding” a calculator performs) when calculating solutions. All rounding should be done at the end of the calculation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \leq |a| < 10$, and n is an integer. Examples:

Problem: Write 6895 in scientific notation. *Answer:* 6.895×10^3

Problem: Write 40,000 in scientific notation. *Answer:* 4×10^4 or 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole number answers should be expressed in their whole number form.

Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.