MATHCOUNTS®

2011
■ School Competition ■
Sprint Round
Problems 1–30

Name _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the right-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

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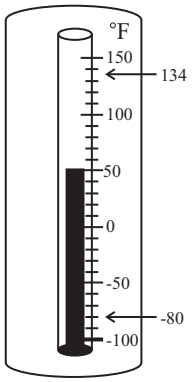
1. What is 25,142 + 13,874 – 3974 expressed to the nearest thousand?

1. _____

2. What is the value of $\frac{1}{2}$ of $2\frac{1}{4}$? Express your answer as a common fraction.

2. _____

3. Death Valley, California has the record for the highest temperature in the U.S. at 134°F. Prospect Creek, Alaska has the coldest recorded temperature at –80°F. How many degrees higher than –80°F is 134°F?



3. _____ °F

4. When Will was 5 years old, his allowance was 25 cents. When he was 6, his allowance was 50 cents. When he was 7, his allowance was $1.00. If Will's allowance continued to double each year, what was his allowance at age 11?

4. $ _____

5. Triangle ABC is similar to triangle XYZ with side AB measuring 4 units, side BC measuring 6 units and side XY measuring 14 units. What is the measure of side YZ?

5. _____ units

6. Ashtyn is $\frac{3}{4}$ as tall as Austin. If Ashtyn is 4.5 feet tall, how tall is Austin?

6. _____ feet

7. The value of $\sqrt{73}$ is between two positive, consecutive integers. What is the product of these two integers?

7. _____

8. If $a \sim b = a^2 + b$, then $3 \sim 5$ is 14 since $3^2 + 5 = 14$. What is the value of $6 \sim 4$?

8. _____

9. The cost of an adult ticket for a Saturday matinee at Turtle Cinema is \$7.00. The cost of a student ticket is \$2.00 less than the cost of an adult ticket. For a particular Saturday matinee, the cinema sold 45 tickets and collected \$265.00. How many student tickets were sold for the Saturday matinee?

9. _____ student tickets

10. The median of a set of 8 consecutive even integers is 61. What is the largest integer in the set?

10. _____

11. The minute hand of a 12-hour clock has rotated 810 degrees since noon. How many minutes have elapsed since noon?

11. _____ minutes

12. In Pascal's Triangle, each number is the sum of the number just above it and to the left and the number just above it and to the right. So the middle number in Row 2 is 2 because $1 + 1 = 2$. What is the sum of the numbers in Row 8 of Pascal's Triangle?

12. _____

Row 0					1					
Row 1				1		1				
Row 2			1		2		1			
Row 3		1		3		3		1		
Row 4	1		4		6		4		1	
Row 5	1		5		10		10		5	1

13. The vertical drop of a roller coaster is the largest difference in height between any high point and the next low point. The vertical drops of five roller coasters at Mandelbrot Amusement Park are shown in the table.

The Parabola	The G Force	The Mean Streak	The Tower of Power	The Maximum Ride
165 feet	119 feet	138 feet	300 feet	198 feet

What is the positive difference between the mean and the median of these values?

13. _____ feet

14. What is the greatest common factor of 5! and $\frac{8!}{3!}$? Express your answer as an integer.

14. _____

15. Three teenagers have integer ages of x, y and z years old. All of the teenagers are between 13 and 19 years old, inclusive. If the product of the three ages is 3705, what is the sum of x, y and z?

15. _____

16. At Jefferson Middle School students either bring lunch or buy lunch. There are 100 sixth grade girls who buy lunch and 70 sixth grade boys who bring lunch. There are 160 boys in the sixth grade and 160 sixth graders who bring lunch. What is the total number of students in the sixth grade at Jefferson Middle School?

16. _____ students

17. Mr. Math commented that his house number is the average of the largest three-digit perfect square and the largest three-digit perfect cube. What is his house number?

17. _____

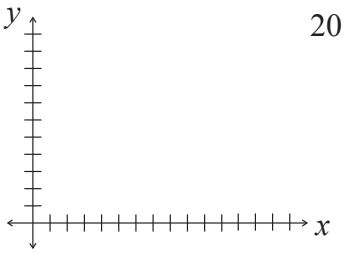
18. Billy Goats invested some money in stocks and bonds. The total amount he invested was $165,000. If he invested 4.5 times as much in stocks as he did in bonds, what was his total investment in stocks?

18. $ _____

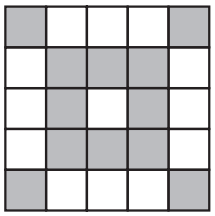
19. A mixture of 40 liters of paint is 25% red tint, 30% yellow tint and 45% water. Five liters of red tint and five liters of yellow tint are added to the original mixture. What is the percent of water in this new mixture?

19. _____ %

20. Circles O and P are tangent to the x-axis at (4, 0) and (13, 0), respectively. Point A(4, 8) is on circle O and point B(13, 10) is on circle P. Line segment OP connects the center of circle O to the center of circle P. What is the slope of segment OP? Express your answer as a common fraction.



20. _____

21. 

A solid 5 × 5 × 5 cube is composed of unit cubes. Each face of the large, solid cube is partially painted with gray paint, as shown. What fraction of the entire solid cube's unit cubes have no paint on them? Express your answer as a common fraction.

21. _____

22. Alice rolls a standard 6-sided die once. Bob rolls a second standard 6-sided die once. Alice wins if the values shown have a positive difference of exactly 1. What is the probability that Alice wins? Express your answer as a common fraction.

22. _____

23. When $0.\overline{21}$ is written as a common fraction in the form $\frac{a}{b}$, what is the value of $b - a$?

23. _____

24. A quarter-circle of radius 3 units is drawn at each of the vertices of a square with sides of 6 units. The area of the shaded region can be expressed in the form $a - b\pi$ square units, where a and b are both integers. What is the value of $a + b$?

24. _____

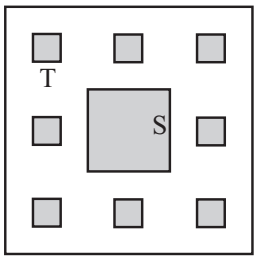
25. The sum of two positive integers is 11, and the positive difference between their squares is 55. What is the positive difference of the two integers?

25. _____

26. How many unit cubes would it take to construct the complete exterior of a hollow cube with edges of 4 units and faces 1 unit thick?

26. _____ unit cubes

27. A square carpet of side length 9 feet is designed with one large shaded square and eight smaller, congruent shaded squares, as shown. If 9:S = S:T = 3 and S and T are the side lengths of the shaded squares, what is the total shaded area?

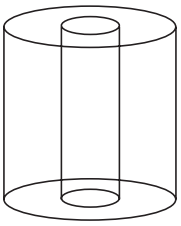


27. _____ sq feet

28. For what base, b, is $14_b + 24_b = 41_b$ true?

28. base _____

29. A hole of radius 2 inches is bored through the center of a right cylinder with radius 6 inches and height 10 inches. What is the total surface area of the resulting solid? Express your answer in terms of π.



29. _____ sq inches

30. When $x^2 - 5x + 3c$ is divided by $x - 3$ the remainder is -12. What is the value of c?

30. _____