MATHCOUNTS®

2011
■ School Competition ■
Countdown Round
Problems 1–60

This section contains problems to be used in the Countdown Round.

The Countdown Round is available as a PowerPoint® file. Please send an e-mail to info@mathcounts.org with "2011 School Competition CDR" in the subject line and indicate the name of the coach and school making the request.

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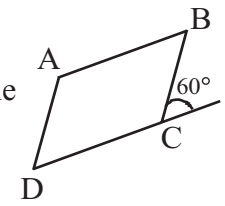
1. Electronics-R-Us sells a popular brand of digital camera for $500. They buy this camera from their dealer for $400. What is the percent increase from the dealer's price to the Electronics-R-Us price?

1. _____ (percent)

2. On a particular map the distance between Hypatia and Euclid is 7.6 cm. If 1 cm represents 35 km, how many kilometers is it from Euclid to Hypatia?

2. _____ (km)

3. Quadrilateral ABCD is a parallelogram. What is the degree measure of angle A?



3. _____ (degrees)

4. What is the greatest common factor of 128, 144 and 480?

4. _____

5. One face of a particular cube has an area of 81 square units. What is the volume of the cube, in cubic units?

5. _____ (cu units)

6. A sequence is formed by adding 2 to the triple of the previous term. If the first term is 1, how many even numbers less than 10,000 would be terms of the sequence?

6. _____ (even numbers)

7. What are the coordinates of point A, if point A has positive integer coordinates and is the same distance from (2, 2) as (2, 2) is from the origin? Express your answer as an ordered pair.

7. _____ (,)

8. The lengths of the sides of a triangle are 6 cm, 7 cm and 9 cm. In a similar triangle whose perimeter is 110 cm, what is the length of the longest side, in centimeters?

8. _____ (cm)

9. What integer that is a multiple of 5 and 7, but not a multiple of 2 or 3, is closest in value to 160?

9. _____

10. In Silverville, the price of a piece of pure silver is directly proportional to its weight. If a two-inch cube ($2 \times 2 \times 2$) of pure silver weighs 3.5 pounds and is worth $480, how many dollars would a five-inch cube ($5 \times 5 \times 5$) be worth in Silverville?

10. _____ (dollars)

11. A haunted house has 7 windows. In how many ways can Frankie the Ghost enter the house by one window and leave by a different window?

11. _____ (ways)

12. In a particular solution weighing 200 grams, 99% of its weight is water. After awhile some water evaporates from the solution but no other liquid loses volume. The remaining solution is now 98% water. How many grams of water evaporated?

12. _____ (grams)

13. How many digits are in the product $(8^3)(5^{12})$ when it is simplified to an integer?

13. _____ (digits)

14. What is the value of the sum $1 - 2 + 3 - 4 + \ldots - 2010 + 2011$?

14. _____

15. Mei Su had 80 coins. She gave most of them to her friends in such a way that each of her friends got at least one coin and no two of her friends got the same number of coins. What is the largest number of friends to whom Mei Su could have given coins?

15. _____ (friends)

16. The diagonal of a square is $13\sqrt{2}$ units. What is the perimeter of the square, in units?

16. _____ (units)

17. What is the largest integer in the solution set of $x^2 - 5x < 20$?

17. _____

18. At Luigi's Pizza Parlor, the ratio of pepperoni pizza slices sold to cheese pizza slices sold is 4:5. In total 225 slices of either cheese or pepperoni pizza were sold. How many pepperoni pizza slices were sold?

18. _____ (slices)

19. Mr. Miller packs his suitcase for a long trip. He packs two different pairs of pants, four different shirts and six ties. How many different outfits can Mr. Miller make with those articles of clothing if an outfit consists of a pair of pants, a shirt and a tie?

19. _____ (outfits)

20. What is the length, in units, of the radius of a circle with an area of $64\pi x^2$ square units? Express your answer in terms of x.

20. _____ (units)

21. In how many ways can you make exactly 40¢ using quarters, dimes and/or nickels? 21. _____ (ways)

22. The value of the sum $\sqrt{10} + \sqrt{17}$ is between two consecutive whole numbers on the number line. What is the product of the two consecutive whole numbers? 22. _____

23. Dad used 30% of the $2\frac{1}{2}$ dozen eggs in the refrigerator. How many eggs did he use? 23. _____ (eggs)

24. If $f(x) = 2x + 1$ and $g(x) = -3$, what is the value of $f(g(x))$? 24. _____

25. The sum of three consecutive odd integers is 249. What is the largest of those three integers? 25. _____

26. Jorge's five cousins are 7, 11, 13, 8 and 6 years of age. What is the positive difference between the mean and the median of their ages, in years? 26. _____ (years)

27. It takes a painter 8 hours to paint 2 rooms. At this rate, how many hours will it take 3 painters to paint 3 rooms? 27. _____ (hours)

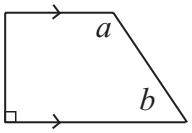
28. Jonathan was born on Monday at 3:15 pm. How many hours old was he at 8:00 am the next day? Express your answer as a mixed number. 28. _____ (hours)

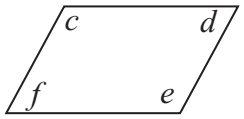
29. What value of x satisfies the equation $\frac{1}{\frac{1}{x}+\frac{1}{2}} = 3$? 29. _____

30. A right, rectangular prism has dimensions $2z$-by-z-by-z units. If the surface area of the prism is 90 square units, what is the value of z? 30. _____

31. What is the largest possible product of two even integers whose sum is 58? 31. _____

32. The degree measures of the angles of a trapezoid and a quadrilateral are represented by a, b, c, d, e and f, as shown in the figures. What is the average of a, b, c, d, e and f, in degrees?





32. _____ (degrees)

33. An item that was originally priced at $20 is now on sale for 30% off. If the sales tax rate is 7.5%, what is the total price Brianna will pay when purchasing this item?

33. _____ (dollars)

34. If $f(x) = 2x - 4$ and $g(x) = 15 - 3x$, what is $f(-1) \times g(-3)$?

34. _____

35. If $3x + 4 = 16$, what is the value of $x^2 + 3x + 2$?

35. _____

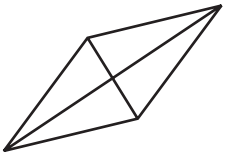
36. The mean of a set of integers is –1. The sum of the integers is –4. How many integers are in the set?

36. _____ (integers)

37. What is the sum of the greatest common factor and least common multiple of 48 and 72?

37. _____

38. A rhombus has diagonals measuring 2 cm and 5 cm. What is its area, in square centimeters?



38. _____ (sq cm)

39. What is the value of $|23 - 17| + |17 - 23|$?

39. _____

40. If 80% of a number N is 256, what is 30% of N?

40. _____

41. In Mr. Davidson's 8th grade class, there are 7 more male students than female students. If $\frac{2}{3}$ of the students are male, how many students are in the class?

41. _____ (students)

42. Let A be the surface area of a 16 units by 1 unit by 1 unit right, rectangular prism, and let B be the surface area of an 8 units by 2 units by 1 unit right, rectangular prism. What is $|A - B|$, in square units?

42. _____ (sq units)

43. If $a \# b = a^b - b^a$, what is the value of $3 \# 5$? 43. _____

44. Each of the ten letters in the word "statistics" is written on a card and placed in a box. If Edwin will choose a card without looking, what is the probability that he will choose a card with a vowel? Express your answer as a common fraction. 44. _____

45. If $4^x + 4^x + 4^x + 4^x = 2^8$, what is the value of x? 45. _____

46. What is the remainder when 1,357,246 is divided by 9? 46. _____

47. How many lines of symmetry does an isosceles right triangle have? 47. _____ (lines)

48. What is the sum of the values of x that satisfy the equation $x^2 - 5x + 5 = 9$? 48. _____

49. What is the value of $(2^3 + 2^4) - 3(2^3)$? 49. _____

50. Pencils are 50¢ a package and notebooks are 35¢ each. Tina must buy at least one package of pencils and one notebook. If she spends the same amount of money on pencils as she does on notebooks, what is the least total amount, in dollars, she will have to spend (not including tax)? Note: The packages of pencils can not be divided. 50. _____ (dollars)

51. What is the maximum area, in square units, of a rectangle whose perimeter is 28 units? 51. _____ (sq units)

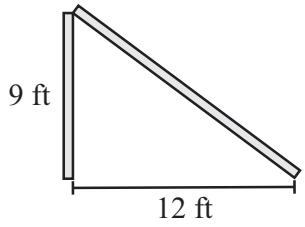
52. How many diagonals, including all space diagonals and face diagonals, does a rectangular prism have? 52. _____ (diagonals)

53. What is one-fourth of the reciprocal of the fraction $\frac{75}{15}$? Express your answer as a common fraction. 53. _____

54. What is the absolute value of the difference between the GCF and the LCM of 6, 8 and 16? 54. _____

55. Regular hexagon ABCDEF is graphed in the coordinate plane. Vertex A is located at (–3, 3), and vertex B is located at (5, 3). What is the perimeter of this hexagon, in units? 55. _____ (units)

56. Lightning struck a pole, breaking it 9 feet from the ground. Its upper section has fallen over and touches the ground 12 feet from the base of the pole. How tall, in feet, was the pole before it was struck by lightning? 56. _____ (feet)



57. If both bases of a trapezoid are tripled and the height is quadrupled, what is the ratio of the area of the original trapezoid to the area of the larger trapezoid? Express your answer as a common fraction. 57. _____

58. Square A measures 8 miles on each side. The area of square B is 8 square miles. What is the positive difference, in square miles, of the areas of square A and square B? 58. _____ (sq miles)

59. If $x = -2$ and $y = -3$, what is the value of $-(-x) + -(-y)$? 59. _____

60. When the following product is simplified, what is its value expressed as a common fraction: $\left(\frac{3}{4}\right)\left(\frac{4}{6}\right)\left(\frac{5}{8}\right)\left(\frac{6}{10}\right)\left(\frac{7}{12}\right)\left(\frac{8}{14}\right)$? 60. _____