MATHCOUNTS®

2007
■ School Competition ■
Sprint Round
Problems 1–30

Name _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the right-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

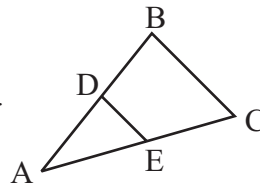| Total Correct	Scorer's Initials

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1. For triangle ABC, points D and E are the midpoints of sides AB and AC, respectively. Side BC measures six inches. What is the measure of segment DE?

1. _____ inches

2. What is the total volume of three boxes if each box is a cube with edge length 4 feet?

2. _____ cu feet

3. What is the least positive integer divisible by the four smallest odd, positive integers?
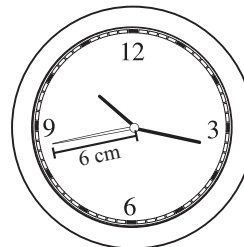
3. _____

4. Jack picks 5 pounds of apples in 8 minutes. Jill picks 6 pounds of apples in 10 minutes. Working together at these rates, how many pounds of apples will they pick in 40 minutes?

4. _____ pounds

5. If the area of a circle is less than 60π square inches, what is the greatest possible integer value of the radius of the circle?

5. _____ inches

6. The second hand on the clock pictured to the right is 6 cm long. How far does the tip of this second hand travel during a period of 30 minutes? Express your answer in terms of π.
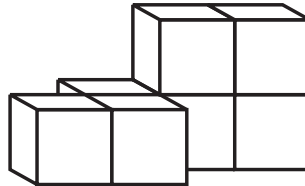
6. _____ centimeters

7. The year 2006 is the most recent calendar year with digits that are each even. What was the most recent year before 1999 with digits that were each even?

7. _____

8. Seven cubic blocks measuring 1 foot on each edge are glued together as shown. What is the total surface area of the resulting solid, including the bottom face?

8. _____ sq feet

9. Five friends have their pictures taken. If they take pictures in groups of three, how many different groups of people can have their picture taken?

9. _____ groups

10. In Mathopolis, an *adult* is a person 21 years of age or older and a *child* is a person under 21 years of age. Exactly half of the adults in Mathopolis are female, and exactly half of the female adults have exactly one biological child. Nobody else has a child, and there are no other children. What percent of the people of Mathopolis are children?

10. _____ percent

11. The difference between two perfect squares is 133. What is the smallest possible sum of the two perfect squares?

11. _____

12. Adam buys oranges at the rate of three oranges for $1. He sells them at the rate of five oranges for $2. In order to make a profit of at least $10, what is the minimum number of oranges he must sell?

12. _____ oranges

13. The length of a rectangle is increased by 25%, but the width of the rectangle is adjusted to keep the area of the rectangle unchanged. By what percent was the rectangle's width adjusted?

13. _____ percent

14. Square A has side lengths each measuring x inches. Square B has side lengths each measuring $4x$ inches. What is the ratio of the area of the smaller square to the area of the larger square? Express your answer as a common fraction.

14. _____

15. What is the largest possible median for the five-member set $\{x, 2x, 3, 2, 5\}$ if x can be any integer?

15. _____

16. If x is a positive integer, what is the value of x for the equation $(x! - (x - 3)!) \div 23 = 1$?

16. _____

17. At Tomi's middle school, 40% of the students are boys. Fifty percent of the boys chose blue as their favorite color, while 20% of the girl students chose blue as their favorite color. Of the students who chose blue as their favorite color, what fraction are boys? Express your answer as a common fraction.

17. _____

18. Line m has the equation $y = 3x + 5$. Line n has the equation $y = kx - 7$. Lines m and n intersect at the point $(-4, -7)$. What is the value of k?

18. _____

19. What is the smallest positive integer that is a perfect square and is also a sum of six consecutive positive integers?

19. _____

20. What is the area of the convex quadrilateral with vertices (−1, 0), (0, 1), (2, 0) and (0, −3)?

20. _____ sq units

21. Point C is on the segment AB which has endpoints A(2, −1) and B(11, 5). Point C is twice as far from point A as it is from point B. What are the coordinates of point C?

21. (____ , ____)

22. How many integers from 100 through 999, inclusive, do not contain any of the digits 2, 3, 4 or 5?

22. _____ integers

23. On Sunday, Jan. 7, 2007, Rob has $5 he saved from his birthday, and he receives his first allowance payment of $35. He will continue to receive a $35 allowance payment every Sunday. However, starting Jan. 7, 2007, Rob is also responsible for paying for his five lunches each week at $3 per lunch and his weekly Saturday movie that costs $9.50. If Rob always adds the remainder of his allowances each week to his current savings, after how many total allowance payments will he have enough money to pay for the $50 CD player he wants and still be able to cover his expenses for the rest of that week?

23. _____ payments

24. When Caitlin starts running towards Asha, Asha is 100 meters from Caitlin. As Caitlin runs, Asha moves directly away from Caitlin at one-third of Caitlin's speed. How far has Caitlin run when she first catches up to Asha?

24. _____ meters

25. At a graduation ceremony, names were read at the rate of five names per minute. If the names had been read at the rate of seven names per minute, the ceremony would have taken 24 minutes less. How many names were read?

25. _____ names

26. An airplane climbs 100 feet during the first second after takeoff. In each succeeding second it climbs 100 feet more than it climbed during the previous second. How many seconds does it take for the plane to reach an altitude of 12,000 feet above its takeoff height?
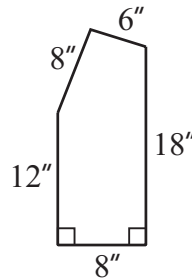
26. _____ seconds

27. When the positive integers are arranged in order, filling in the successive diagonals of an infinite grid from top to bottom, as shown, the integer 41 is in the (5, 5) spot. What integer would we see in the (10, 10) spot if the rest of the grid were visible?

1	2	4	7	11
3	5	8	12	17
6	9	13	18	24	•••
10	14	19	25	32
15	20	26	33	41

⋮

27. _____

28. What is the area of the pentagon shown?

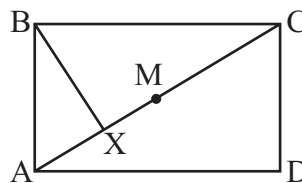28. _____ sq inches

29. We know the following to be true:
     1. Z and K are integers with $500 < Z < 1000$ and $K > 1$;
     2. $Z = K \times K^2$.
What is the value of K for which Z is a perfect square?

29. _____

30. In the figure shown, a perpendicular segment is drawn from B in rectangle ABCD to meet diagonal AC at point X. Side AB is 6 cm and diagonal AC is 10 cm. How far is point X from the midpoint M of the diagonal AC? Express your answer as a decimal to the nearest tenth.

30. _____ centimeters