MATHCOUNTS®

2007
■ School Competition ■
Countdown Round
Problems 1–60

This section contains problems to be used in the Countdown Round.

The Countdown Round is available as a PowerPoint® file. Please send an e-mail to info@mathcounts.org with "2007 School Competition CDR" in the subject line and indicate the name of the coach and school making the request.

National Sponsors

Lockheed Martin * Raytheon Company
Texas Instruments * Northrop Grumman Foundation
National Society of Professional Engineers * 3M Foundation
General Motors Foundation * CNA Foundation
National Aeronautics and Space Administration

LOCKHEED MARTIN

MATHCOUNTS
National Competition Sponsor
2006-2008

Founding Sponsors: National Society of Professional Engineers, National Council of Teachers of Mathematics and CNA Foundation

1. Sandy plans to paint one wall in her bedroom. The wall is 9 feet high and 12 feet long. There is a 2-foot by 4-foot area on that wall that she will not have to paint due to the window. How many square feet will she need to paint? 1. _____

2. Square tiles measuring 4 inches on each side will be used to tile a 1-foot by 1-foot area. How many of these tiles are needed? 2. _____

3. Starting with the third term in the sequence 1, 5, 6, 11, 17, …, each term is the sum of the previous two terms in the sequence. What is the seventh term in the sequence? 3. _____

4. The function $f(x)$ is defined by $f(x) = x^2 - x$. What is the value of $f(4)$? 4. _____

5. Half of 1% of 40 is one less than what number? Express your answer as a decimal to the nearest tenth. 5. _____

6. Cory made a complete list of the prime numbers between 1 and 25. What is the sum of the smallest prime number and the largest prime number on his list? 6. _____

7. Simplify this expression to a common fraction: $\dfrac{1}{\frac{1}{\left(\frac{1}{2}\right)^1} + \frac{1}{\left(\frac{1}{2}\right)^2} + \frac{1}{\left(\frac{1}{2}\right)^3}}$ 7. _____

8. Allergy shots cost Kyle \$3 each. If he gets one shot twice per calendar month, how many dollars does it cost for one year of shots? 8. _____

9. Two shaded quarter-circles, each with a radius of 4 units, are formed in the 4 by 8 rectangular region shown. What is the area of the non-shaded region, in square units? Express the area in the form $a - b\pi$ where a and b are integers. 9. _____

10. A grocery bag can hold up to 8 pounds of potatoes. What is the least number of bags needed to hold 50 pounds of potatoes? 10. _____

11. Julie eats 3 granola bars every 24 hours while Kirsten eats 2 granola bars every 12 hours. What percent fewer granola bars per hour does Julie eat? 11. _____

12. The prime factorization of 2007 is $3^2 \times 223$. How many ordered pairs of positive integers (x, y) satisfy the equation $xy = 2007$? 12. _____

13. The diagonal of a square is $\sqrt{2}$ inches long. How many inches long is the side of the square? 13. _____

14. The earth rotates at a constant rate and rotates 360 degrees in one day. How many degrees does it rotate in one hour? 14. _____

15. Alicia receives $\frac{2}{5}$ of a \$120 gift, and Jason receives $\frac{1}{3}$ of the gift. What is the ratio of Jason's share to Alicia's share? Express your answer as a common fraction. 15. _____

16. In his pocket Sam had \$4 composed solely of quarters and nickels. He had a total of 20 coins. How many nickels did he have? 16. _____

17. What is the sum of the four positive factors of the positive integer value of $\sqrt{196}$? 17. _____

18. In how many ways can four people sit in a row of five chairs? 18. _____

19. Each edge of a cube has length 3 inches. What is the number of square inches in the cube's total surface area? 19. _____

20. One-sixth of one-third of one-fourth of what number is equal to five? 20. _____

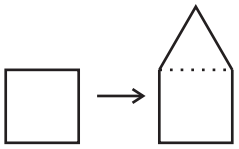
21. What is the sum of the tens digit and the ones digit of the integer form of $(2 + 3)^{23}$? 21. _____

22. What is the smallest positive perfect square that is divisible by both 2 and 3? 22. _____

23. How many perfect squares less than 100 have a tens digit of 6? 23. _____

24. Bill buys a stock that decreases by 20% on the first day, and then on the second day the stock increases by 30% of its value at the end of the first day. What was the overall percent increase in Bill's stock over the two days? 24. _____

25. A square is changed into a pentagon by building an equilateral triangle on top of it, as shown. By what percent does the perimeter increase going from the square to the pentagon? 25. _____



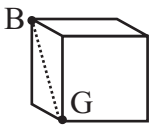
26. What is the value of $\frac{a}{b} - \frac{b}{a}$ if $a = 5$ and $b = 2$? Express your answer as a decimal to the nearest tenth. 26. _____

27. Jackson records the number of people inside each of the first 100 cars he sees that can carry up to four people inside. If 22 cars carry exactly 4 people, 18 carry exactly 3 people, and 29 carry exactly 2 people, how many cars have only 1 person inside? 27. _____

28. If $x@y = xy - 2x$, what is the value of $(5@3) - (3@5)$? 28. _____

29. What is the simplified value of $(7^4)(8-2^3)+(11^{4(8)-32})$? 29. _____

30. The figure shown is a cube. The distance between vertices B and G is $5\sqrt{2}$ units. What is the volume of the cube, in cubic units? 30. _____



31. Two rockets fly directly at each other. One travels at a speed of 8500 miles per hour, and the other travels at a speed of 3500 miles per hour. How many miles apart will they be one minute before they collide? 31. _____

32. What is the smallest prime that is 10 less than a perfect square? 32. _____

33. One-half of one-seventh of T equals one-third of one-fifth of 90. What is the value of T? 33. _____

34. What is the value of $(8 - 4)! \div (8 - 3)!$? Express your answer as a common fraction. 34. _____

35. A right isosceles triangle has a hypotenuse with measure 18 centimeters. What is the length, in centimeters, of one leg of the triangle? Express your answer is simplest radical form. 35. _____

36. What is the smallest multiple of 6 greater than 115? 36. _____

37. Jo is thinking of a positive integer less than 100. It is one less than a multiple of 8, and it is three less than a multiple of 7. What is the greatest possible integer Jo could be thinking of? 37. _____

38. What is the value of the expression $2^4 + 2^4 + 2^4 + 2^4$? 38. _____

39. In 36 years Adam's age will be 2.5 times his current age. How old is he now? 39. _____

40. How many more cents per item is "3 items for \$10" than "2 items for \$5?" Express your answer to the nearest whole number. 40. _____

41. What is the value of y in the equation $\frac{30}{50} = \sqrt{\frac{y}{50}}$? 41. _____

42. How many integer side lengths are possible to complete a triangle in which the other sides measure 6 units and 3 units? 42. _____

43. What is the remainder when 2007 is divided by 25? 43. _____

44. The perimeter of a rectangle of width 6 feet is 10 yards. In square feet, what is the area of the rectangle? 44. _____

45. What is the value of $f(-1)$ if $f(x) = x^2 - 2x$? 45. _____

46. The radius of a circle is 2 inches. When the radius is doubled, by how many square inches is the area increased? Express your answer in terms of π. 46. _____

47. Karen drove continuously from 9:40 a.m. until 1:20 p.m. of the same day and covered a distance of 165 miles. What was her average speed in miles per hour? 47. _____

48. Kyle, Kate and Kendall split the cost of a pizza evenly between the three of them. If Kyle paid $3.24, what was the total cost of the pizza? 48. _____

49. According to a nursery rhyme recipe, 24 blackbirds were to be baked in a pie. But due to a blackbird shortage, the baker cut the recipe to make the pie filling three-fourths the normal volume. How many fewer blackbirds were needed? 49. _____

50. If the area of a square is 36 square units, then half the square's perimeter is how many units? 50. _____

51. A digital music library contains 12 days of music and takes up 16,000 megabytes of disk space. On average, how many megabytes of disk space does one hour of music in this library take up? Express your answer to the nearest whole number. 51. _____

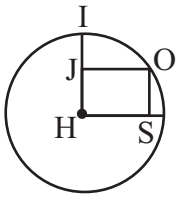
52. Ken wants to fence in a square region that shares a side with his house, so only three sides of the square will be made by the fence. The area of the fenced-in region measures 900 square feet. How many linear feet of fencing does he need? 52. _____

53. Karla has only dimes and quarters in her purse. The value of her dimes is exactly five times the value of her quarters. What is the least number of coins she could have in her purse if there is at least one quarter? 53. _____

54. When $0.\overline{36}$ is expressed as a common fraction, what is the sum of the numerator and denominator? 54. _____

55. Gina has five pairs of white socks, three pairs of black socks and two pairs of red socks. What percent of her socks are red? 55. _____

56. In this figure, points O and I are on the circle with center H and HI = 12 units. What is the length, in units, of diagonal JS in rectangle JOSH?



56. _____

57. A play has three acts that are 80 minutes, 50 minutes and 45 minutes long, respectively. There are also two 15-minute intermissions. If the play starts at 5:30 p.m., at what time does it end?

57. _____

58. What is the value of the expression $-7 + 3 \times 2^3 - 6$?

58. _____

59. In how many ways can four people line up in a straight line if the youngest person cannot be first in line?

59. _____

60. What is the simplified value of $\frac{\sqrt{24}}{\sqrt{30}} \div \frac{\sqrt{20}}{3\sqrt{25}}$?

60. _____