



Mission Math Utah

# Spring Competition

## Middle School Division: Multiple Choice Test

March 16, 2019

Name: \_\_\_\_\_

### General Information

- **Do not open this test until you are instructed to do so.**
- This section contains **30 multiple choice questions**. You will have exactly **40 minutes** to work on them.
- Each question is followed by five answer choices. Only one of the answer choices is correct. On the provided answer form, circle the letter of the answer you think is correct.
- Electronic devices, including calculators, must be turned off.
- You may do any work you want on this test, and you may keep the test once you are finished.

### Grading

- Each correct answer will be worth 1 point.
- Each incorrect or blank answer will be worth 0 points.
- Only answers marked on the provided answer form will be scored.

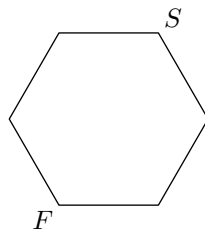
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These problems are meant to be challenging. Don't worry if you are unable to solve a problem. Try to focus on the problems you think you may be able to solve. You are not penalized for guessing, so **take an educated guess on any problems you are not able to solve**. If you finish before the time is up, use the remaining time to check your work.

1. What is the value of  $19 \times 3 + 11 \times 3$ ?

- (A) 24      (B) 36      (C) 90      (D) 798      (E) 1881

2. Damir and Joao each walk around a park in the shape of a hexagon. Each side of the park is one mile long. They both start at point  $S$  and finish at point  $F$ . Damir walks clockwise, beginning to the right and down, while Joao walks counter clockwise, beginning to the left. Who walks further, and by how many miles?



- (A) Damir walks two more miles than Joao.  
(B) Damir walks one more mile than Joao.  
(C) Joao walks one more mile than Damir.  
(D) Joao walks two more miles than Damir.  
(E) Damir and Joao walk the same number of miles.

3. Together, Jupiter and Saturn have 141 moons, while Jupiter has 17 more moons than Saturn. How many moons does Saturn have?

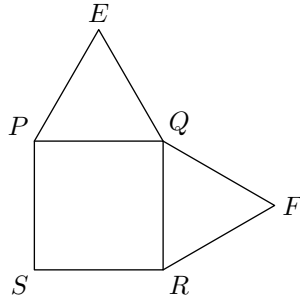
- (A) 52      (B) 62      (C) 69      (D) 78      (E) 79

4. It takes Glenn 30 minutes to walk 2 miles from his house to his school. At the same rate, how many minutes would it take him to walk from his house to the park, which is 3 miles away?

- (A) 20      (B) 30      (C) 40      (D) 45      (E) 60

5. A motor shop has 21 bicycles, 13 unicycles, and 7 four wheelers. How many wheels are there in the shop in total?
- (A) 41      (B) 62      (C) 63      (D) 83      (E) 87
6. It takes Penelope 55 minutes to get ready for school and 10 minutes to walk to school. If Penelope wants to be at school at 7:40, at what time should she wake up?
- (A) 6:15      (B) 6:25      (C) 6:35      (D) 6:45      (E) 7:05
7. Teddy is facing north. He turns 90 degrees to the left, then 270 degrees to the right, then another 180 degrees. What direction is Teddy facing now?
- (A) North      (B) South      (C) East  
(D) West      (E) Cannot be determined
8. If two fair coins are flipped, what is the probability of getting exactly 1 head and 1 tail?
- (A)  $\frac{1}{16}$       (B)  $\frac{1}{8}$       (C)  $\frac{1}{4}$       (D)  $\frac{1}{2}$       (E) 1
9. Allison takes half the candy out of a bag, and Beatrice takes a third of the remaining candy. If there are 4 pieces of candy in the bag now, how many were there to start with?
- (A) 8      (B) 10      (C) 12      (D) 16      (E) 24
10. Mingus makes a cake in the shape of a cylinder that has a volume of  $72\pi$  cubic centimeters. It has a height of 2 centimeters. What is the cake's radius, in centimeters?
- (A) 3      (B) 6      (C) 9      (D) 12      (E) 36

11. Points  $E$  and  $F$  lie outside square  $PQRS$  so that  $\triangle PQE$  and  $\triangle QRF$  are equilateral triangles. What is the degree measure of angle  $EQF$ ?



- (A)  $90^\circ$     (B)  $105^\circ$     (C)  $120^\circ$     (D)  $135^\circ$     (E)  $150^\circ$
12. Andy has 5 black socks, 8 white socks, and 2 red socks in a drawer. If he pulls out socks randomly, how many socks does he have to pull out to guarantee a matching pair?
- (A) 2    (B) 3    (C) 4    (D) 8    (E) 12
13. Mount Tinga is currently 10,000 feet tall. Its height increases by 20% every two years. How many feet tall will Mount Tinga be in 4 years?
- (A) 12,000    (B) 14,000    (C) 14,400  
(D) 18,000    (E) 20,736
14. At Raheem Middle School, 30 students are in the math club, 40 students are in the chess club, and 80 students are in neither. If Raheem Middle School has 136 students, how many students are in both the math club and the chess club?
- (A) 7    (B) 14    (C) 22    (D) 26    (E) 46

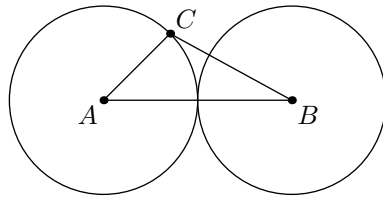
15. In a bag, there are 10 blue marbles, 5 red marbles with blue dots, and 7 red marbles with green dots. If a red marble is drawn randomly, what is the probability that it has a green dot?

(A)  $\frac{5}{22}$     (B)  $\frac{7}{22}$     (C)  $\frac{5}{12}$     (D)  $\frac{7}{12}$     (E)  $\frac{7}{10}$

16. The function  $f(n)$  is defined as  $f(n) = (-1)^n(n + 1)$ . What is the value of  $f(f(f(1)))$ ?

(A)  $-2$     (B)  $-1$     (C)  $0$     (D)  $1$     (E)  $2$

17. Circles with centers  $A$  and  $B$  both have radius 1 and are externally tangent. Point  $C$  is on the circle with center  $A$  such that  $\angle CAB = 45^\circ$ . What is the area of  $\triangle ABC$ ?

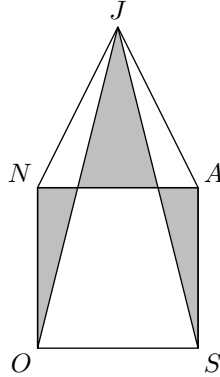


(A)  $\frac{1}{4}$     (B)  $\frac{\sqrt{2}}{2}$     (C)  $\frac{3}{4}$     (D)  $1$     (E)  $\sqrt{2}$

18. Last year, Peter was three times Amy's age. Next year, Peter will be twice Amy's age. In how many years will Amy be 90 percent of Peter's age?

(A) 9    (B) 17    (C) 18    (D) 33    (E) 34

19. Pentagon  $JASON$  consists of square  $SONA$  and triangle  $JAN$ , as shown. If the height from  $J$  to  $\overline{NA}$  is equal to the side length of  $SONA$ , what is the ratio of the area of self-intersecting pentagon  $JONAS$ , shaded in the figure, to the area of  $JASON$ ?



- (A)  $\frac{1}{4}$     (B)  $\frac{1}{3}$     (C)  $\frac{3}{8}$     (D)  $\frac{4}{9}$     (E)  $\frac{1}{2}$
20. What is the maximum number of points of intersection when a line, a circle and a regular pentagon are drawn in the same plane?
- (A) 7    (B) 10    (C) 14    (D) 17    (E) 28
21. When  $m$  gold coins are divided evenly between 11 leprechauns, there are  $n$  coins left over. When  $m^2$  coins are shared among the leprechauns, there are  $\frac{n}{2}$  left over. What is  $n$ ?
- (A) 2    (B) 4    (C) 6    (D) 8    (E) 10
22. A river flows at 6 miles per hour. If it takes a boat, moving at a constant speed relative to the water, 6 hours to travel a certain distance upstream, and 3 hours to travel that same distance downstream, what is the boat's speed in miles per hour?
- (A) 8    (B) 9    (C) 12    (D) 16    (E) 18

23. 5 friends stand in a row from shortest to tallest. Carol is taller than Dan but shorter than Eric. Alice is taller than Bob, and there is 1 person between her and Eric. Bob is next to Dan. Who is in the middle of the line?

(A) Alice    (B) Bob    (C) Carol    (D) Dan    (E) Eric

24. The five answer choices for this problem have both a mean and range of 50. Out of all sets of five (not necessarily distinct) integers that have a mean and range of 50, how many values are possible for the largest of the five integers?

(A) 30    (B) 31    (C) 54    (D) 55    (E) 80

25. If

$$16^{16^{16}} = 4^{4^x} = 2^{2^y},$$

what is  $x+y$ ? (Note:  $a^{b^c}$  should be read as  $a^{(b^c)}$ , not  $(a^b)^c$ . Hence  $3^{3^3} = 3^{27}$ .)

(A) 93.5    (B) 94.5    (C) 97.5    (D) 98.5    (E) 640

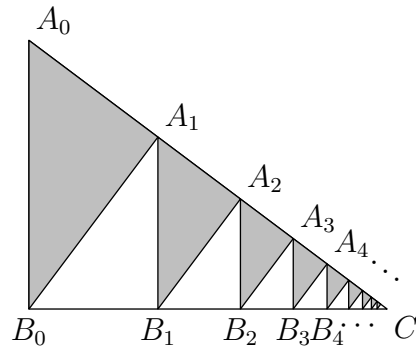
26. Rudy and Ricky each randomly choose one of the numbers 0.5, 1.5, 2.5, ..., 9.5, 10.5. Their numbers are not necessarily distinct. What is the probability that the product of their numbers is 0.25 more than an integer?

(A)  $\frac{59}{121}$     (B)  $\frac{60}{121}$     (C)  $\frac{1}{2}$     (D)  $\frac{61}{121}$     (E)  $\frac{62}{121}$

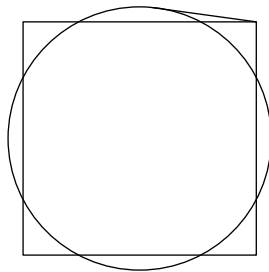
27. The numbers 1, 2, ..., 9 are randomly placed into the 9 squares of a  $3 \times 3$  grid. What is the probability that the sum of the numbers in each row of the grid is the same?

(A)  $\frac{1}{140}$     (B)  $\frac{1}{135}$     (C)  $\frac{1}{126}$     (D)  $\frac{1}{120}$     (E)  $\frac{1}{112}$

28. In right triangle  $A_0B_0C$  with right angle  $B_0$ ,  $A_0B_0 = 3$  and  $B_0C = 4$ . For each positive integer  $n$ , point  $A_n$  is the foot of the altitude from  $B_{n-1}$  to  $\overline{A_0C}$ , and point  $B_n$  is the foot of the altitude from  $A_n$  to  $\overline{B_0C}$ . All triangles  $A_nB_nA_{n+1}$  for nonnegative integers  $n$  are shaded. What is the area of the shaded region?



- (A)  $\frac{10}{3}$     (B)  $\frac{24}{7}$     (C)  $\frac{150}{41}$     (D)  $\frac{96}{25}$     (E) 6
29. For a real number  $r$ , the *fractional part* of  $r$  is defined to be the difference between  $r$  and the greatest integer less than or equal to  $r$ , and is denoted by  $\{r\}$ . For instance,  $\{4.128\} = 0.128$ ,  $\{\pi\} = 0.14159\dots$ ,  $\{\sqrt{3}\} = 0.732\dots$ , and  $\{6\} = 0$ . For how many integers  $n$  between 1 and 100 inclusive is  $0 \leq \{\sqrt{n}\} \leq \frac{1}{2}$ ?
- (A) 40    (B) 45    (C) 50    (D) 55    (E) 60
30. A circle with radius 2 and a square are drawn with the same center so that the area inside the circle and outside the square is equal to the area inside the square and outside the circle. A line segment begins at a vertex of the square and is tangent to the circle, ending at the point of tangency as shown. Which of the following is closest to the square of the length of this segment?



- (A) 1.5    (B) 1.75    (C) 2    (D) 2.25    (E) 2.5